EXAMPLE 1.25

Find the overall transfer function of the system whose signal flow graph is shown in fig 1.

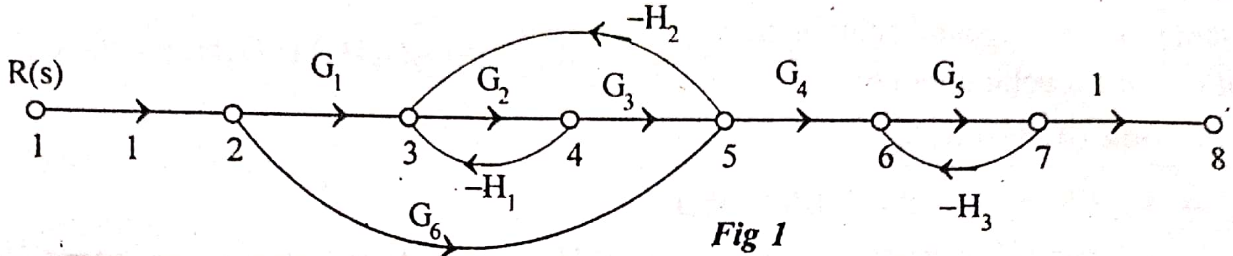

Fig 1

SOLUTION

I. Forward Path Gains

There are two forward paths. $\therefore$ **K = 2**

Let forward path gains be P_1 and P_2 .

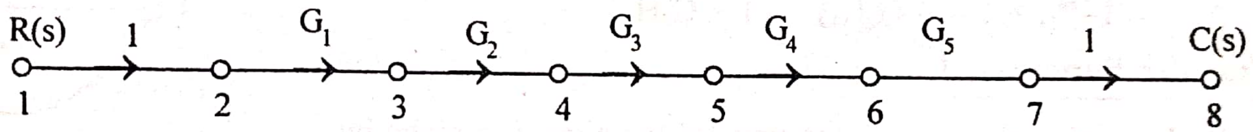

Fig 2 : Forward path-1.

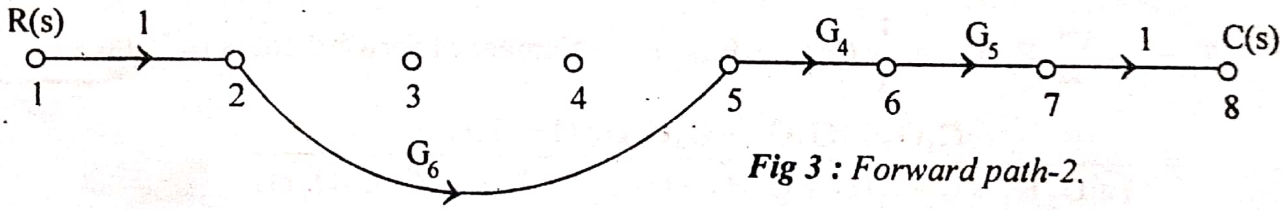

Fig 3 : Forward path-2.

Gain of forward path-1, $P_1 = G_1\ G_2\ G_3\ G_4\ G_5$

Gain of forward path-2, $P_2 = G_4G_5G_6$

II. Individual Loop Gain

There are three individual loops. Let individual loop gains be P_{11}, P_{21} and P_{31}.

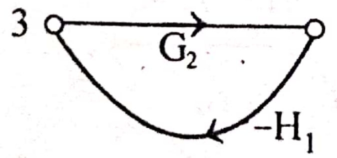

Fig 4 : Loop-1.

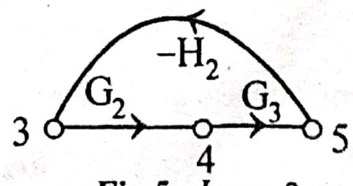

Fig 5 : Loop-2.

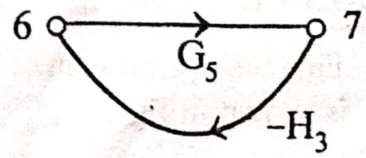

Fig 6 : Loop-3.

Loop gain of individual loop-1, $P_{11} = -G_2\ H_1$

Loop gain of individual loop-2, $P_{21} = -G_2\ G_3\ H_2$

Loop gain of individual loop-3, $P_{31} = -G_5\ H_3$

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III. Gain Products of Two Non-touching Loops

There are two combinations of two non-touching loops. Let the gain products of two non touching loops be P_{12} and P_{22}.

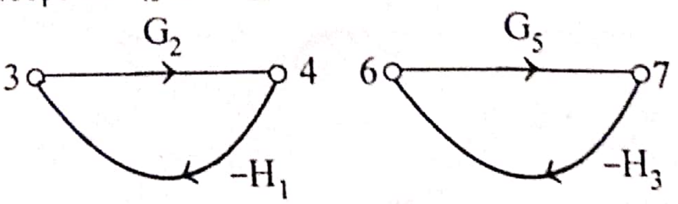

Fig 7 : First combination of 2 non-touching loops.

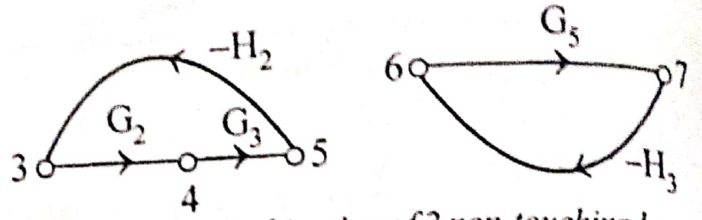

Fig 8 : Second combination of 2 non-touching loops.

Gain product of first combination of two non touching loops } $P_{12} = P_{11}P_{31} = (-G_2H_1)(-G_5H_3) = G_2G_5H_1H_3$

Gain product of second combination of two non touching loops } $P_{22} = P_{21}P_{31} = (-G_2G_3H_2)(-G_5H_3) = G_2G_3G_5H_2H_3$

IV. Calculation of Δ and Δ_K

$$\Delta = 1 - (P_{11} + P_{21} + P_{31}) + (P_{12} + P_{22})$$

$$= 1 - (-G_2H_1 - G_2G_3H_2 - G_5H_3) + (G_2G_5H_1H_3 + G_2G_3G_5H_2H_3)$$

$$= 1 + G_2H_1 + G_2G_3H_2 + G_5H_3 + G_2G_5H_1H_3 + G_2G_3G_5H_2H_3$$

$\Delta_1 = 1$, Since there is no part of graph which is not touching with first forward path.

The part of the graph which is non touching with second forward path is shown in fig 9.

$\Delta_2 = 1 - P_{11} = 1 - (-G_2H_1) = 1 + G_2H_1$

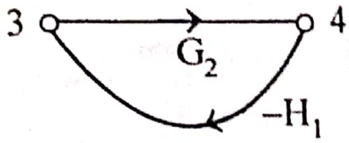

Fig 9

V. Transfer Function, T

By Mason's gain formula the transfer function, T is given by,

$$T = \frac{1}{\Delta}\sum_K P_K\Delta_K = \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2) \quad \text{(Number of forward paths is 2 and so K = 2)}$$

$$= \frac{G_1G_2G_3G_4G_5 + G_4G_5G_6(1+G_2H_1)}{1+G_2H_1+G_2G_3H_2+G_5H_3+G_2G_5H_1H_3+G_2G_3G_5H_2H_3}$$

$$= \frac{G_1G_2G_3G_4G_5 + G_4G_5G_6 + G_2G_4G_5G_6H_1}{1+G_2H_1+G_2G_3H_2+G_5H_3+G_2G_5H_1H_3+G_2G_3G_5H_2H_3}$$

$$= \frac{G_2G_4G_5\left[G_1G_3 + G_6/G_2 + G_6H_1\right]}{1+G_2H_1+G_2G_3H_2+G_5H_3+G_2G_5H_1H_3+G_2G_3G_5H_2H_3}$$

EXAMPLE 1.26

Find the overall gain of the system whose signal flow graph is shown in fig 1.

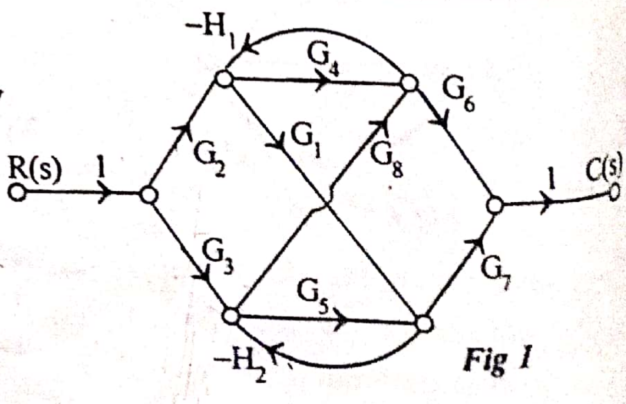

Fig 1

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SOLUTION

Let us number the nodes as shown in fig 2.

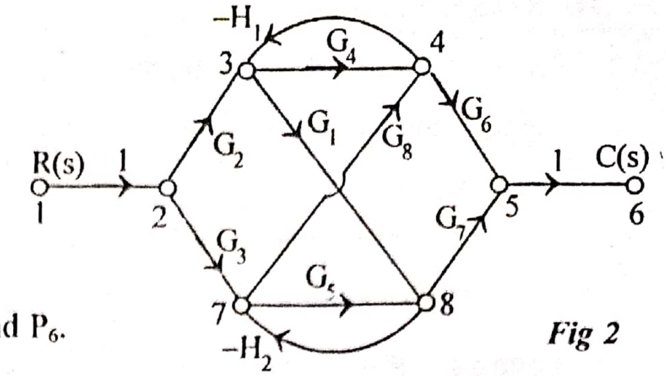

Fig 2

I. Forward Path Gains

There are six forward paths. $\therefore K = 6$

Let the forward path gains be P_1, P_2, P_3, P_4, P_5 and P_6.

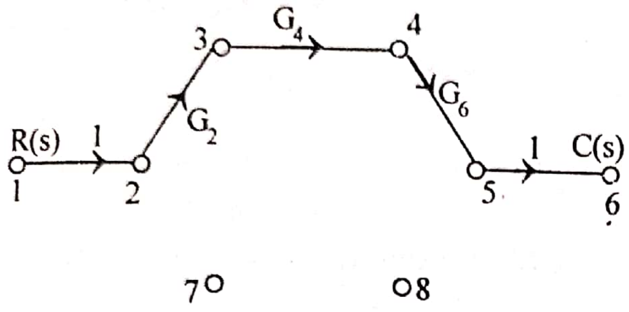

Fig 3 : Forward path-1.

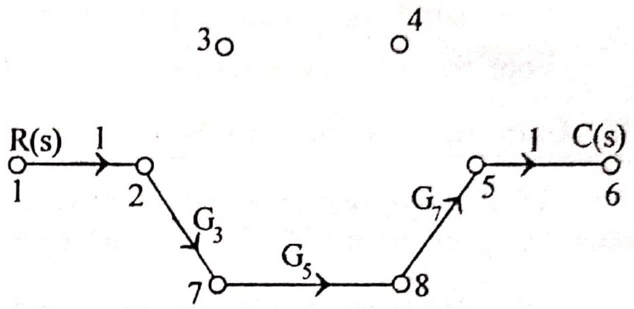

Fig 4 : Forward path-2.

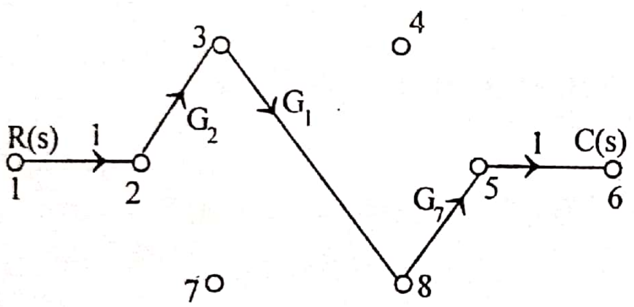

Fig 5 : Forward path-3.

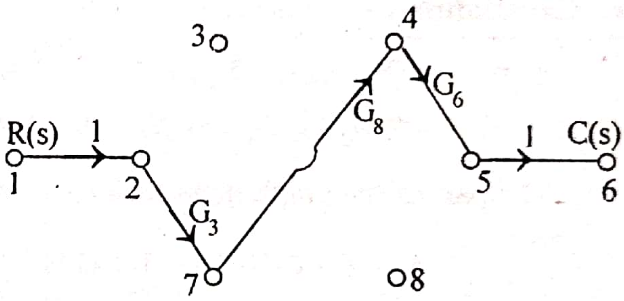

Fig 6 : Forward path-4.

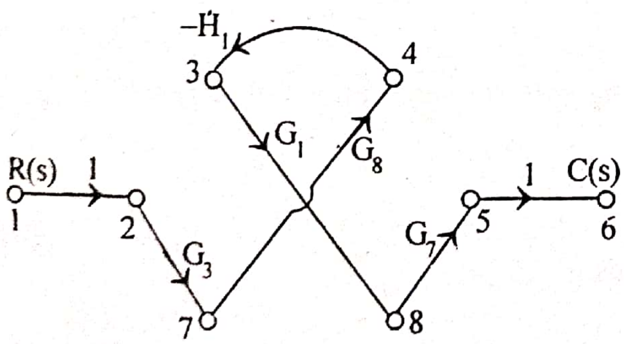

Fig 7 : Forward path-5.

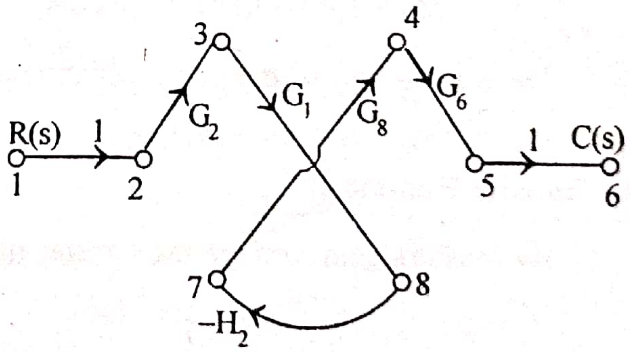

Fig 8 : Forward path-6.

Gain of forward path-1, $P_1 = G_2\,G_4\,G_6$

Gain of forward path-2, $P_2 = G_3\,G_5\,G_7$

Gain of forward path-3, $P_3 = G_1\,G_2\,G_7$

Gain of forward path-4, $P_4 = G_3\,G_8\,G_6$

Gain of forward path-5, $P_5 = -G_1G_3G_7G_8H_1$

Gain of forward path-6, $P_6 = -G_1G_2G_6G_8H_2$

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II. Individual Loop Gain

There are three individual loops.

Let individual loop gains be P_{11}, P_{21} and P_{31}.

Fig 9 : Loop-1.

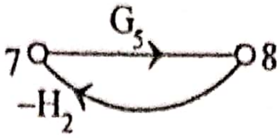

Fig 10 : Loop-2.

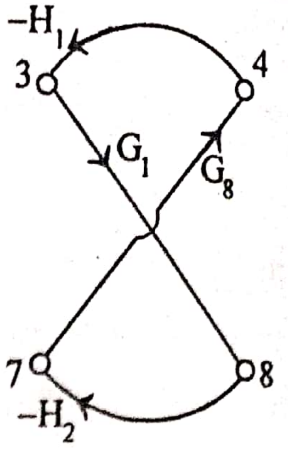

Fig 11 : Loop-3.

Loop gain of individual loop-1, $P_{11} = -G_4H_1$

Loop gain of individual loop-2, $P_{21} = -G_5H_2$

Loop gain of individual loop-3, $P_{31} = G_1G_8H_1H_2$

III. Gain Products of Two Non-touching Loops

There is only one combination of two non-touching loops. Let gain product of two non-touching loops be P_{12}.

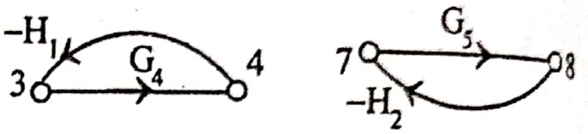

Fig 12 : Combination of 2 non-touching loo...

Gain product of first combination of two non - touching loops } $P_{12} = P_{11}P_{21} = (-G_4H_1)(-G_5H_2) = G_4G_5H_1H_2$

IV. Calculation of Δ and Δ_K

$$\Delta = 1-(P_{11}+P_{21}+P_{31})+P_{12} = 1-(-G_4H_1-G_5H_2+G_1G_8H_1H_2)+G_4G_5H_1H_2$$
$$= 1+G_4H_1+G_5H_2-G_1G_8H_1H_2+G_4G_5H_1H_2$$

The part of the graph non-touching forward path – 1 is shown in fig 13.

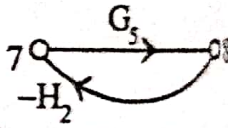

Fig 13

$$\therefore \Delta_1 = 1-(-G_5H_2) = 1+G_5H_2$$

The part of the graph non-touching forward path –2 is shown in fig 14.

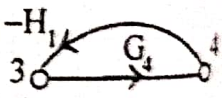

Fig 14

$$\therefore \Delta_2 = 1-(-G_4H_1) = 1+G_4H_1$$

There is no part of the graph which is non-touching with forward paths 3, 4, 5 and 6.

$$\therefore \Delta_3 = \Delta_4 = \Delta_5 = \Delta_6 = 1$$

V. Transfer Function, T

By Mason's gain formula the transfer function, T is given by,

$$T = \frac{1}{\Delta}\left(\sum_K P_K \Delta_K\right) \quad \text{(Number of forward paths is six and so } K = 6)$$

$$= \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4 + P_5\Delta_5 + P_6\Delta_6)$$

$$= \frac{G_2G_4G_6(1+G_5H_2)+G_3G_5G_7(1+G_4H_1)+G_1G_2G_7+G_3G_6G_8 - G_1G_3G_7G_8H_1 - G_1G_2G_6G_8H_2}{1+G_4H_1+G_5H_2-G_1G_8H_1H_2+G_4G_5H_1H_2}$$

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EXAMPLE 1.27

Find the overall gain C(s)/R(s) for the signal flow graph shown in fig 1.

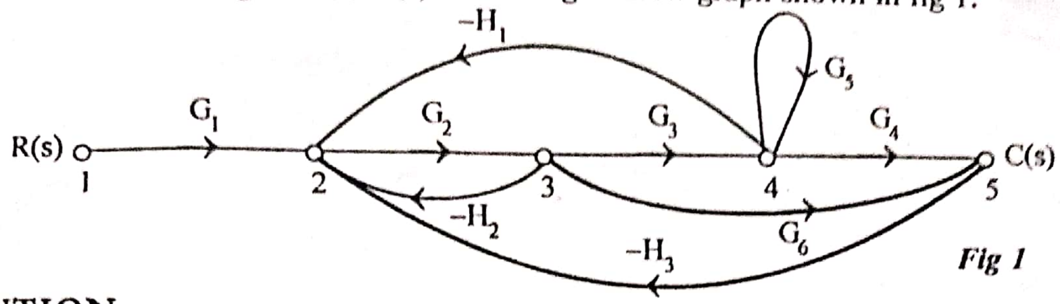

Fig 1

SOLUTION

I. Forward Path Gains

There are two forward paths. $\therefore K = 2$. Let the forward path gains be P_1 and P_2.

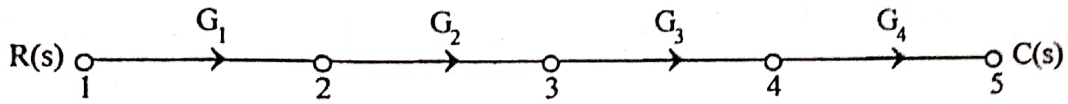

Fig 2 : Forward path-1.

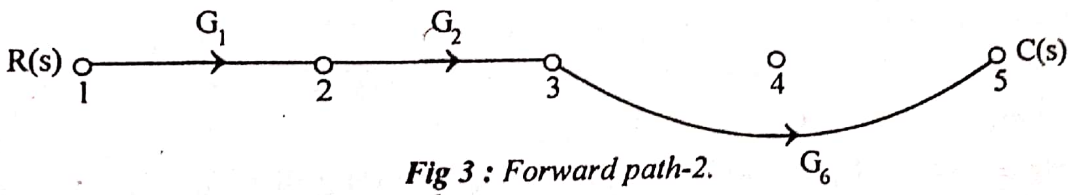

Fig 3 : Forward path-2.

Gain of forward path-1, $P_1 = G_1G_2G_3G_4$

Gain of forward path-2, $P_2 = G_1G_2G_6$

II. Individual Loop Gain

There are five individual loops. Let the individual loop gains be p_{11}, p_{21}, p_{31}, p_{41} and p_{51}.

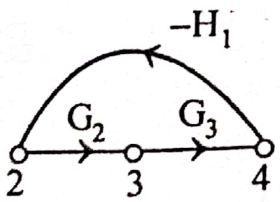

Fig 4 : loop-1.

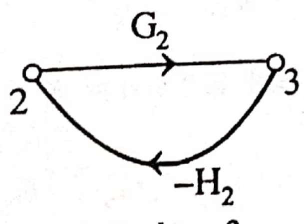

Fig 5 : loop-2.

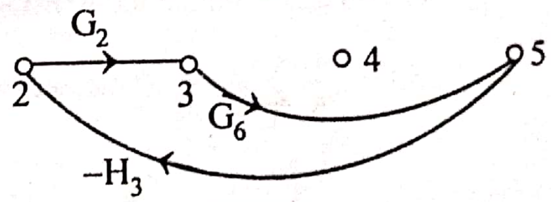

Fig 6 : loop-3.

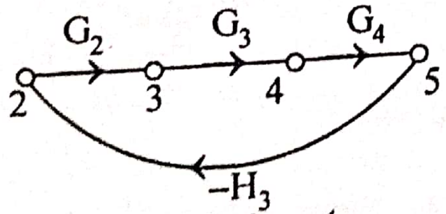

Fig 7 : loop-4.

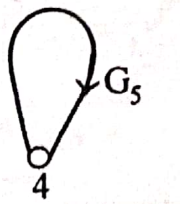

Fig8 : loop-5.

Loop gain of individual loop-1, $P_{11} = -G_2G_3H_1$

Loop gain of individual loop-2, $P_{21} = -H_2G_2$

Loop gain of individual loop-3, $P_{31} = -G_2G_6H_3$

Loop gain of individual loop-4, $P_{41} = -G_2G_3G_4H_3$

Loop gain of individual loop-5, $P_{51} = G_5$

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II. Gain Products of Two Non-touching Loops

There are two combinations of two non-touching loops.

Let the gain products of two non-touching loops be P_{12} and P_{22}.

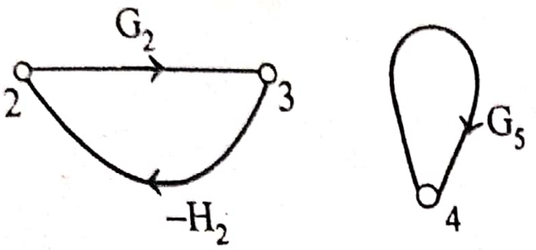

Fig 9 : First combination of two non-touching loops.

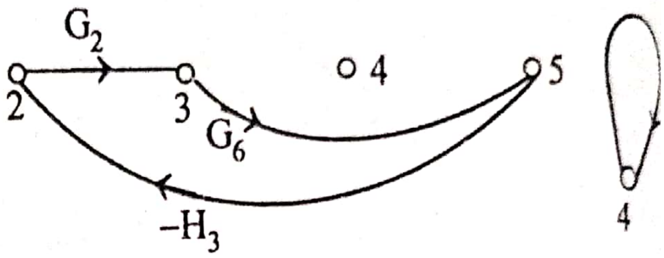

Fig 10 : Second combination of two non-touching lo

Gain product of first combination of two non touching loops $\left.\right\}$ $P_{12} = P_{21}P_{51} = (-G_2H_2)(G_5) = -G_2G_5H_2$

Gain product of second combination of two non touching loops $\left.\right\}$ $P_{22} = P_{31}P_{51} = (-G_2G_6H_3)(G_5) = -G_2G_5G_6H_3$

IV. Calculation of Δ and Δ_K

$$\Delta = 1 - (P_{11} + P_{21} + P_{31} + P_{41} + P_{51}) + (P_{12} + P_{22})$$
$$= 1 - (-G_2G_3H_1 - H_2G_2 - G_2G_3G_4H_3 + G_5 - G_2G_6H_3)$$
$$+ (-G_2H_2G_5 - G_2G_5G_6H_3)$$

Since there is no part of graph which is not touching forward path-1, $\Delta_1 = 1$.

The part of graph which is not touching forward path-2 is shown in fig 11.

$\therefore \Delta_2 = 1 - G_5$

Fig 11

V. Transfer Function, T

By Mason's gain formula the transfer function, T is given by,

$$T = \frac{1}{\Delta}\sum_K P_K\Delta_K \text{ (Number of forward path is 2 and so K = 2)}$$

$$= \frac{1}{\Delta}[P_1\Delta_1 + P_2\Delta_2] = \frac{1}{\Delta}[G_1G_2G_3G_4 \times 1 + G_1G_2G_6(1 - G_5)]$$

$$= \frac{G_1G_2G_3G_4 + G_1G_2G_6 - G_1G_2G_5G_6}{1 + G_2G_3H_1 + H_2G_2 + G_2G_3G_4H_3 - G_5 + G_2G_6H_3 - G_2H_2G_5 - G_2G_5G_6H_3}$$

EXAMPLE 1.28

Find the overall gain C(s)/R(s) for the signal flow graph shown in fig 1.

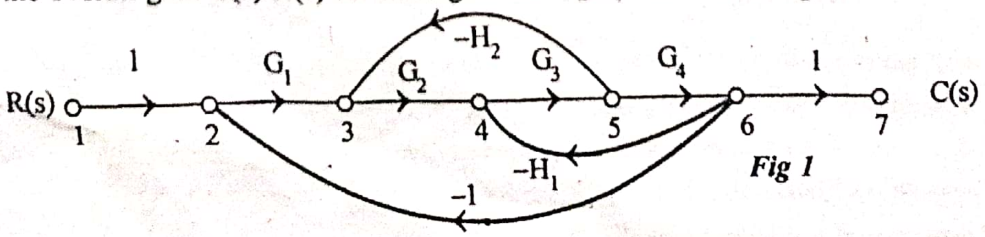

Fig 1

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SOLUTION

I. Forward Path Gains

There is only one forward path. $\therefore K = 1$.
Let the forward path gain be P_1.

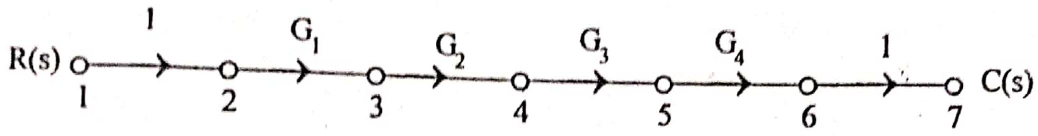

Fig 2 : Forward path-1.

Gain of forward path-1, $P_1 = G_1G_2G_3G_4$

II. Individual Loop Gain

There are three individual loops. Let the loop gains be P_{11}, P_{21}, P_{31}.

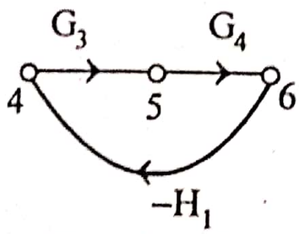

Fig 3 : loop-1.

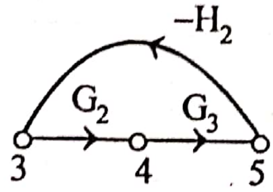

Fig 4 : loop-2.

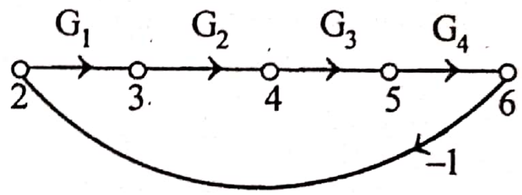

Fig 5 : loop-3.

Loop gain of individual loop-1, $P_{11} = -G_3G_4H_1$
Loop gain of individual loop-2, $P_{21} = -G_2G_3H_2$
Loop gain of individual loop-3, $P_{31} = -G_1G_2G_3G_4$

III. Gain Products of Two Non-touching Loops

There are no possible combinations of two non-touching loops, three non-touching loops, etc.

IV. Calculation of Δ and Δ_K

$$\Delta = 1-(P_{11}+P_{21}+P_{31})$$
$$= 1-(-G_3G_4H_1 - G_2G_3H_2 - G_1G_2G_3G_4)$$
$$= 1+G_3G_4H_1 + G_2G_3H_2 + G_1G_2G_3G_4$$

Since no part of the graph is non-touching with forward path-1, $\Delta_1 = 1$.

V. Transfer Function, T

By Mason's gain formula the transfer function, T is given by,

$$T = \frac{C(s)}{R(s)} = \frac{1}{\Delta}\sum_K P_K\Delta_K = \frac{1}{\Delta}P_1\Delta_1 \text{ (Number of forward path is 1 and so K = 1)}$$

$$= \frac{G_1G_2G_3G_4}{1+G_3G_4H_1+G_2G_3H_2+G_1G_2G_3G_4}$$

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EXAMPLE 1.29

The signal flow graph for a feedback control system is shown in fig 1. Determine the closed loop transfer function C(s)/R(s).

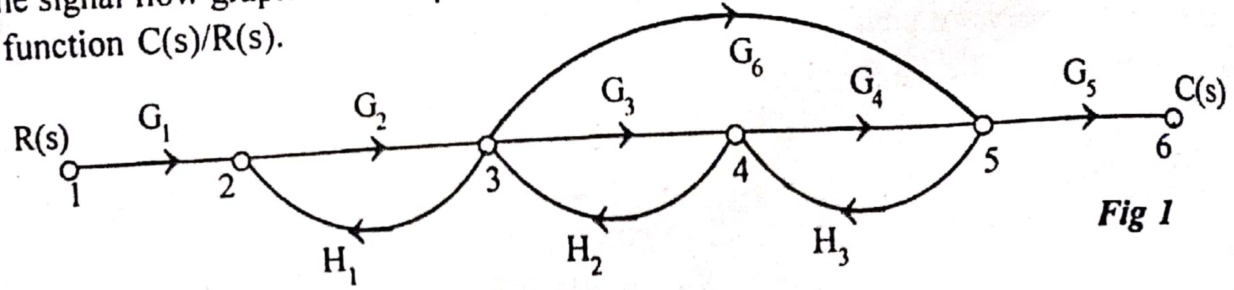

Fig 1

SOLUTION

I. Forward Path Gains

There are two forward paths. $\therefore K = 2$.

Let forward path gains be P_1 and P_2.

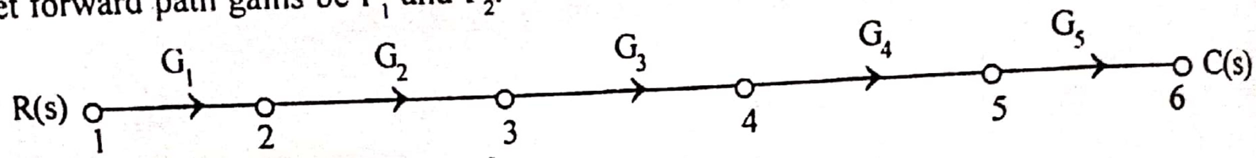

Fig 2 : Forward path-1.

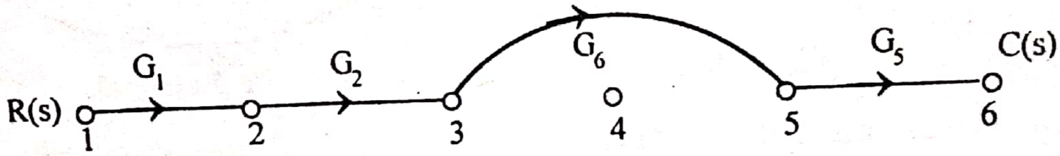

Fig 3: Forward path-2.

Gain of forward path-1, $P_1 = G_1 G_2 G_3 G_4 G_5$

Gain of forward path-2, $P_2 = G_1 G_2 G_6 G_5$

II. Individual Loop Gain

There are four individual loops. Let individual loop gains be P_{11}, P_{21}, P_{31} and P_{41}.

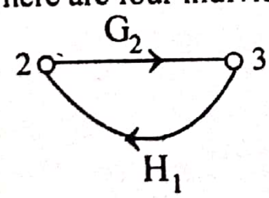

Fig 4 : loop-1.

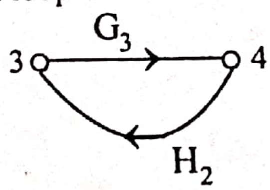

Fig 5 : loop-2.

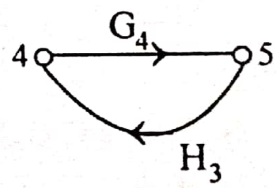

Fig 6 : loop-3.

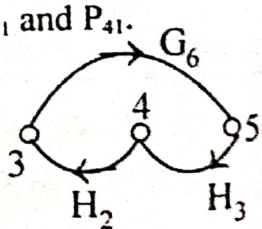

Fig 7 : loop-4.

Loop gain of individual loop-1, $P_{11} = G_2H_1$

Loop gain of individual loop-2, $P_{21} = G_3H_2$

Loop gain of individual loop-3, $P_{31} = G_4H_3$

Loop gain of individual loop-4, $P_{41} = G_6H_2H_3$

III. Gain Products of Two Non-touching Loops

There is only one combination of two non-touching loops. Let the gain products of two non-touching loops be P_{12}.

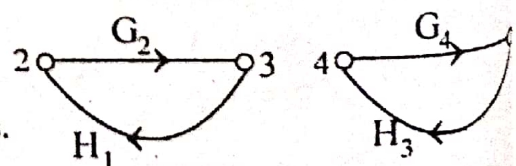

Fig 8 : First combination of two non touching loops

Gain product of first combination of two non - touching loops $\Big\}$ $P_{12} = (G_2H_1)(G_4H_3)$

$$= G_2G_4H_1H_3$$

IV. Calculation of Δ and Δ_K

$$\Delta = 1-(P_{11}+P_{21}+P_{31}+P_{41})+P_{12}$$
$$= 1-(G_2H_1+G_3H_2+G_4H_3+G_6H_2H_3)+G_2G_4H_1H_3$$
$$= 1-G_2H_1-G_3H_2-G_4H_3-G_6H_2H_3+G_2G_4H_1H_3$$

Since there is no part of graph which is non-touching with forward path-1 and.2,

$$\Delta_1 = \Delta_2 = 1$$

V. Transfer Function, T

By Mason's gain formula the transfer function. T is given by,

$$T = \frac{1}{\Delta}\sum_K P_K\Delta_K = \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2) \quad \text{(Number of forward paths is two and so } K=2\text{)}$$

$$= \frac{G_1G_2G_3G_4G_5 + G_1G_2G_5G_6}{1-G_2H_1-G_3H_2-G_4H_3-G_6H_2H_3+G_2G_4H_1H_3}$$

EXAMPLE 1.30

Convert the given block diagram to signal flow graph and determine C(s)/R(s).

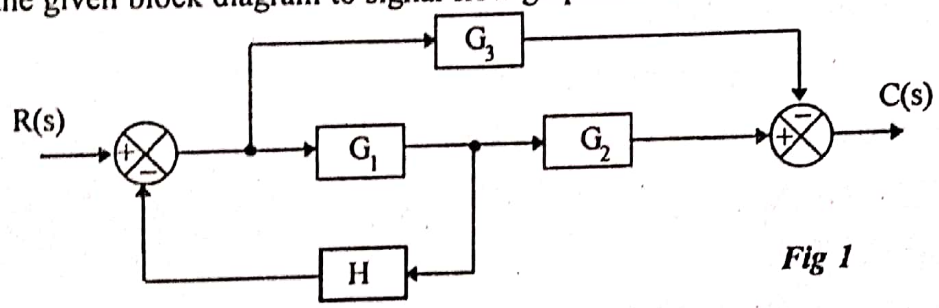

Fig 1

SOLUTION

The nodes are assigned at input, output, at every summing point & branch point as shown in fig 2.

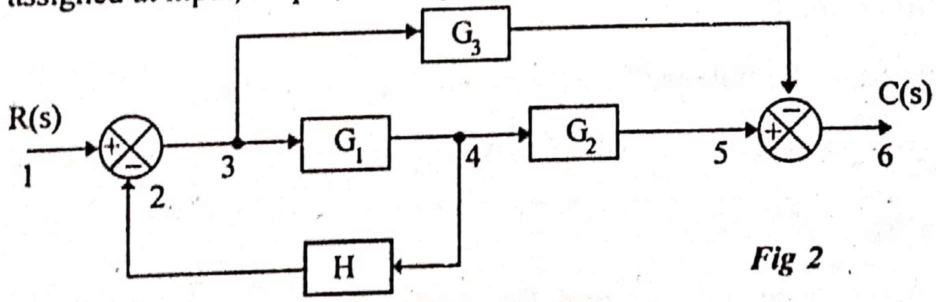

Fig 2

The signal flow graph of the above system is shown in fig 3.

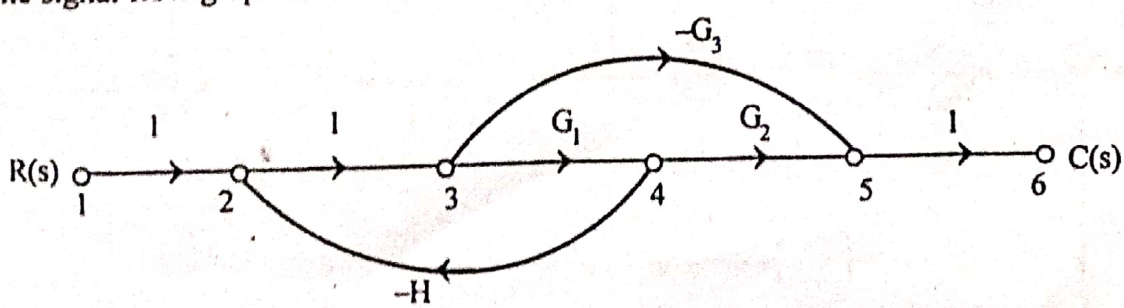

Fig 3

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Forward Path Gains

There are two forward paths. $\therefore K=2$

Let the forward path gains be P_1 and P_2.

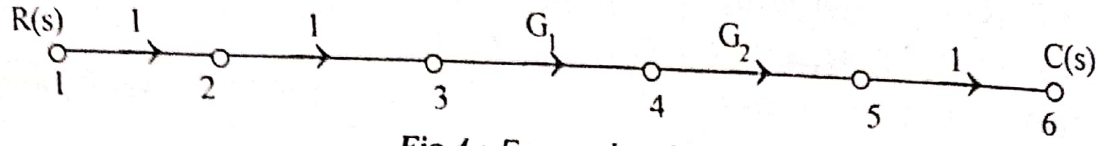

Fig 4 : Forward path-1.

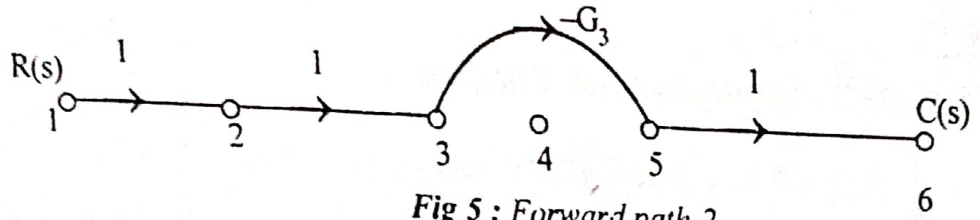

Fig 5 : Forward path-2.

Gain of forward path-1, $P_1 = G_1G_2$

Gain of forward path-2, $P_2 = -G_3$

II. Individual Loop Gain

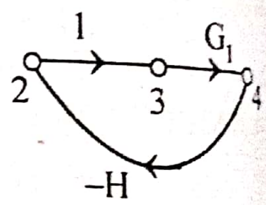

Fig 6 : loop-1.

There is only one individual loop. Let the individual loop gain be P_{11}.

Loop gain of individual loop-1, $P_{11} = -G_1H$.

III. Gain Products of Two Non-touching Loops

There are no combinations of non-touching Loops.

IV. Calculation of Δ and Δ_K

$$\Delta = 1 - [P_{11}] = 1 + G_1H$$

Since there are no part of the graph which is non-touching with forward path-1 and 2,

$$\Delta_1 = \Delta_2 = 1$$

V. Transfer Function, T

By Mason's gain formula the transfer function, T is given by,

$$T = \frac{1}{\Delta}\sum_K P_K\Delta_K = \frac{1}{\Delta}[P_1\Delta_1 + P_2\Delta_2] = \frac{G_1G_2 - G_3}{1+G_1H}$$

EXAMPLE 1.31

Convert the block diagram to signal flow graph and determine the transfer function using Mason's gain formula.

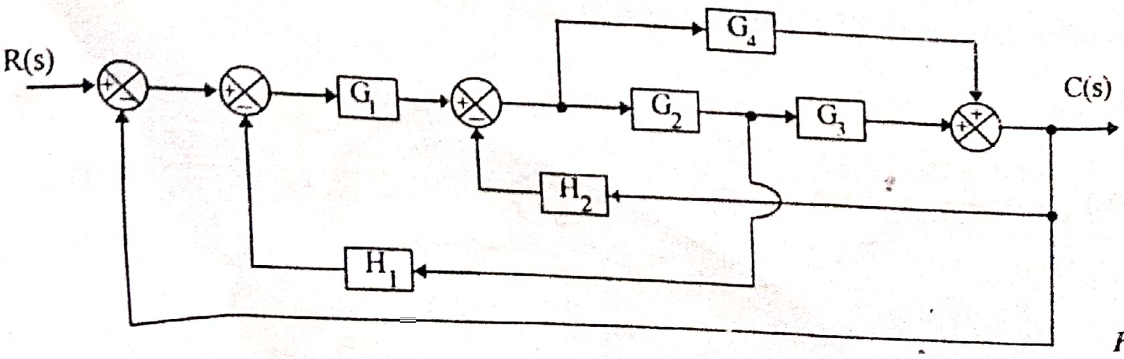

Fig 1

SOLUTION

The nodes are assigned at input, ouput, at every summing point & branch point as shown in fig 2.

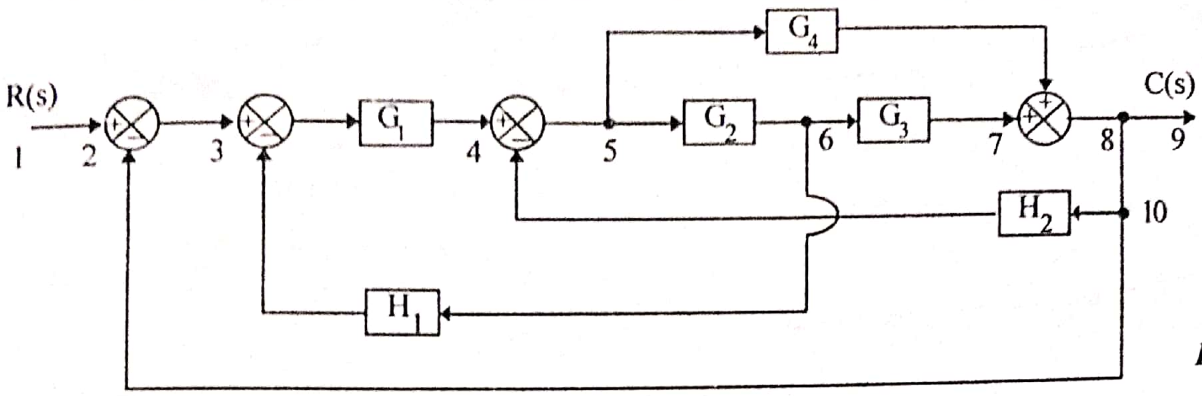

Fig 2

The signal flow graph for the above block diagram is shown in fig 3.

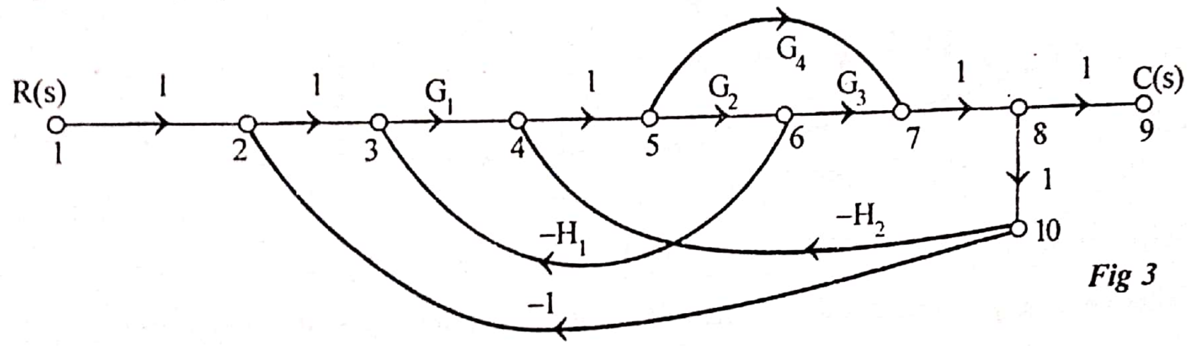

Fig 3

I. Forward Path Gains

There are two forward paths. $\therefore K=2$.

Let the gain of the forward paths be P_1 and P_2.

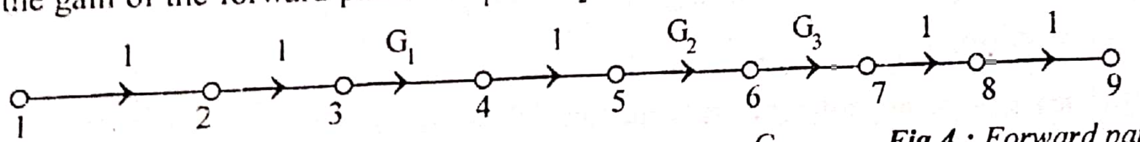

Fig 4 : Forward path-1.

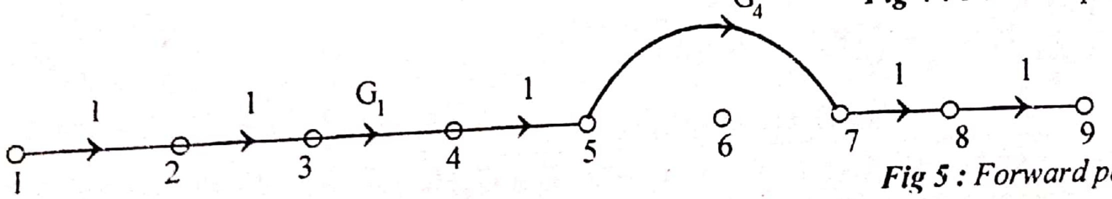

Fig 5 : Forward path-2.

Gain of forward path-1, $P_1 = G_1G_2G_3$

Gain of forward path-2, $P_2 = G_1G_4$

II. Individual Loop Gain

There are five individual loops. Let the individual loop gain be P_{11}, P_{21}, P_{31}, P_{41} and P_{51}.

Loop gain of individual loop-1, $P_{11} = -G_1G_2G_3$

Loop gain of individual loop-2, $P_{21} = -G_2G_1H_1$

Loop gain of individual loop-3, $P_{31} = -G_2G_3H_2$

Loop gain of individual loop-4, $P_{41} = -G_1G_4$

Loop gain of individual loop-5, $P_{51} = -G_4H_2$

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EXAMPLE 1.16

Reduce the block diagram shown in fig 1 and find C/R.

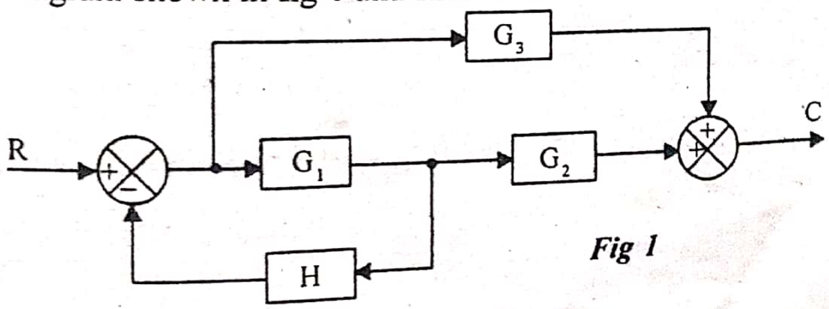
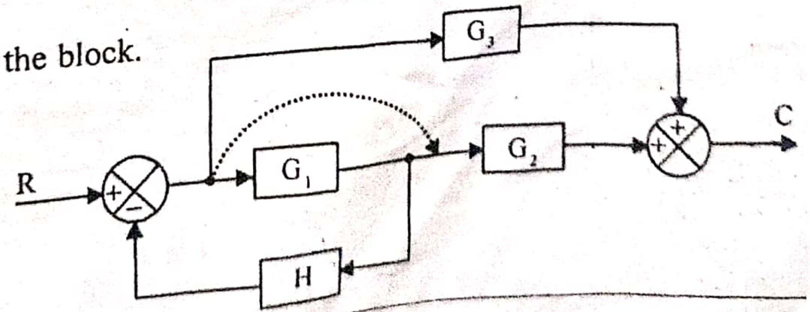


Fig 1

SOLUTION

Step 1: Move the branch point after the block.



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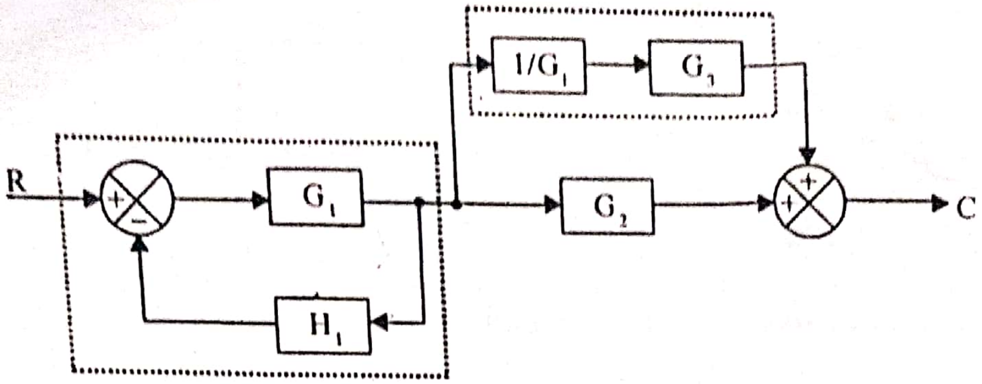
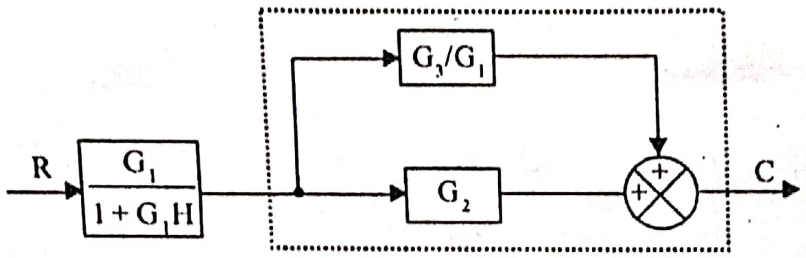
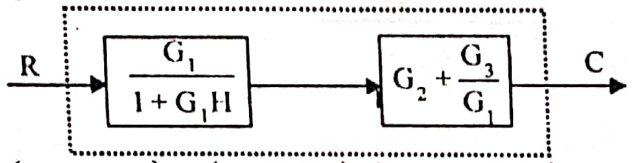
Step 2: Eliminate the feedback path and combining blocks in cascade.



Step 3: Combining parallel blocks



Step 4: Combining blocks in cascade



$$\frac{C}{R}=\left(\frac{G_1}{1+G_1H}\right)\left(G_2+\frac{G_3}{G_1}\right)=\left(\frac{G_1}{1+G_1H}\right)\left(\frac{G_1G_2+G_3}{G_1}\right)=\frac{G_1G_2+G_3}{1+G_1H}$$

RESULT

The overall transfer function of the system, $\frac{C}{R}=\frac{G_1G_2+G_3}{1+G_1H}$

EXAMPLE 1.17

Using block diagram reduction technique find closed loop transfer function of the system whose block diagram is shown in fig 1.

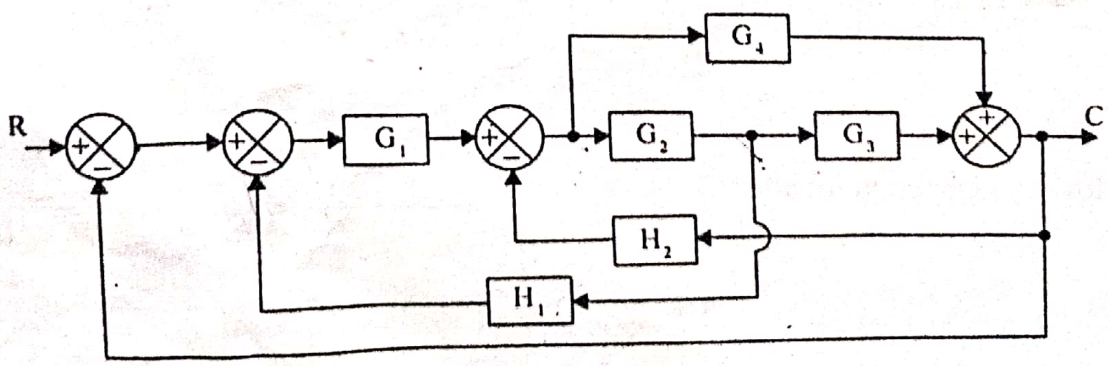
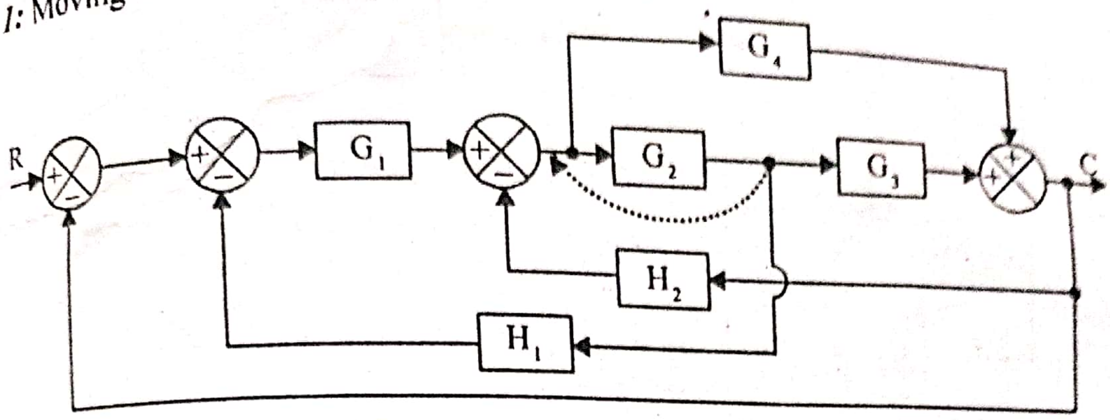


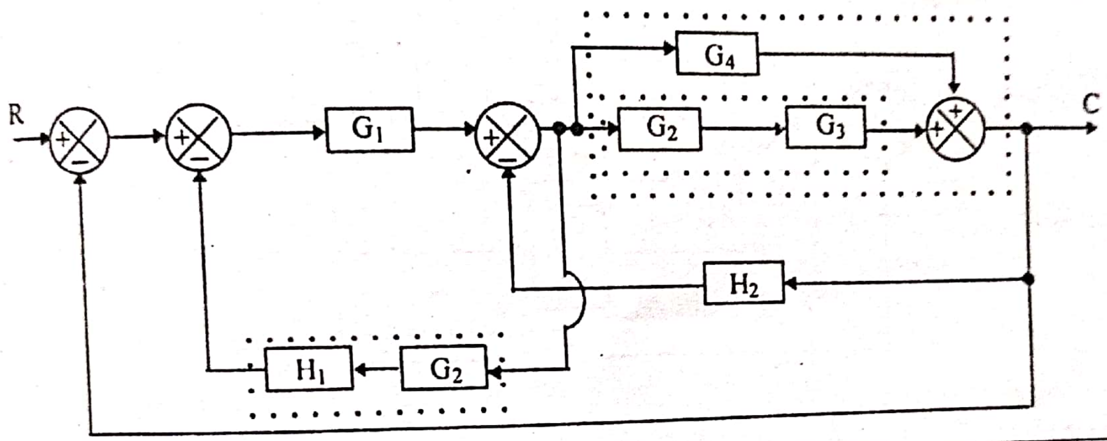
Fig 1

SOLUTION

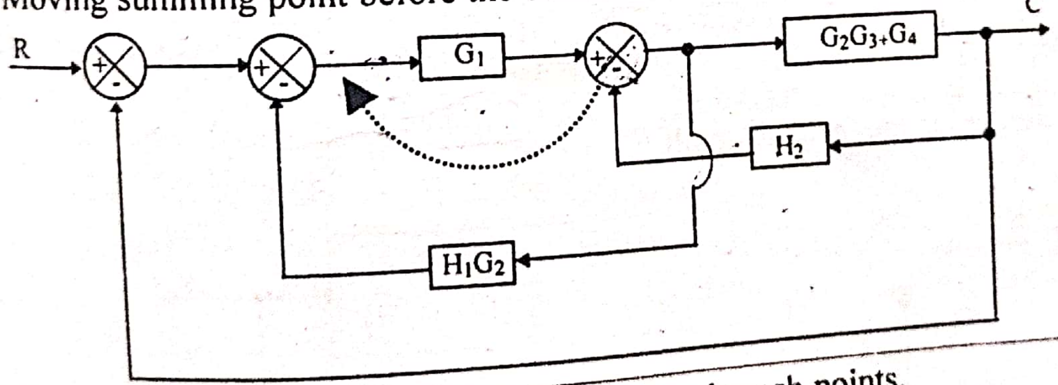
Step 1: Moving the branch point before the block



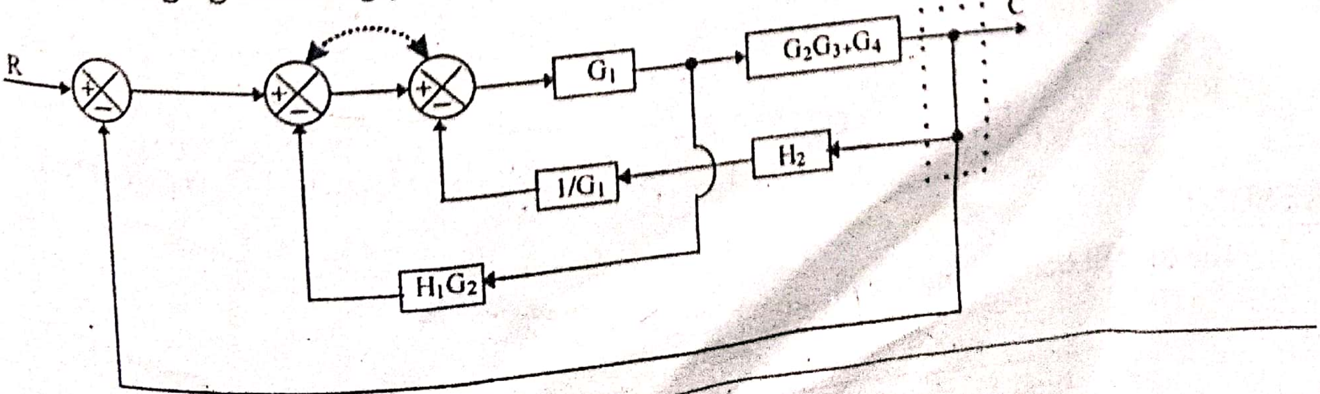
Step 2: Combining the blocks in cascade and eliminating parallel blocks



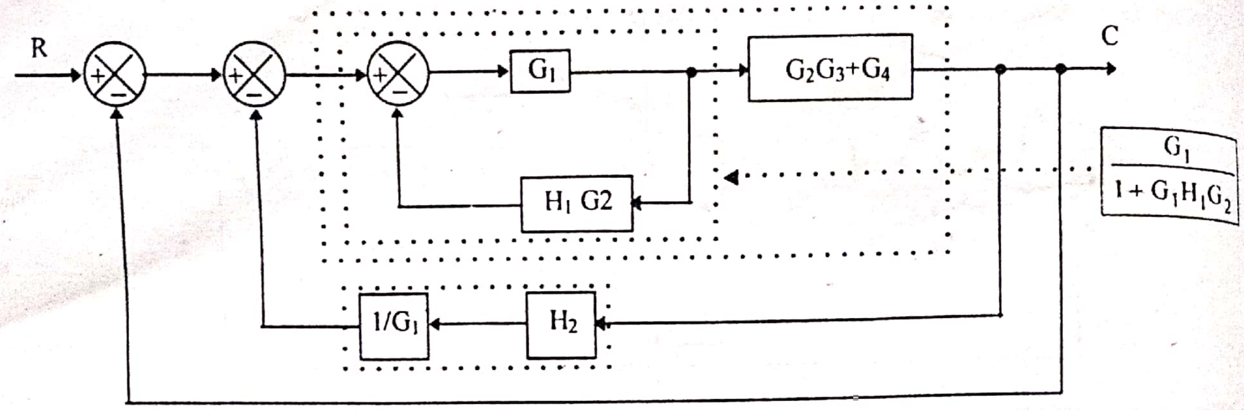
Step 3: Moving summing point before the block.



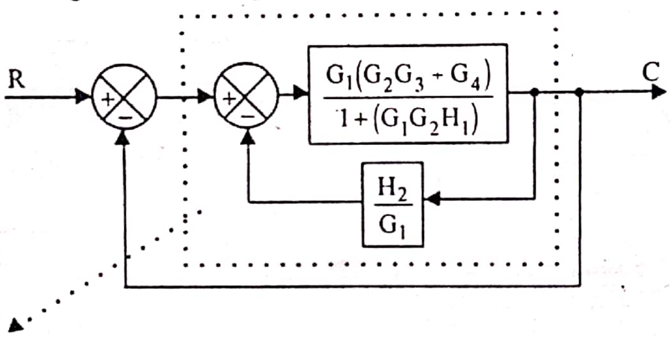
Step 4: Interchanging summing points and modifying branch points.



Step 5: Eliminating the feedback path and combining blocks in cascade

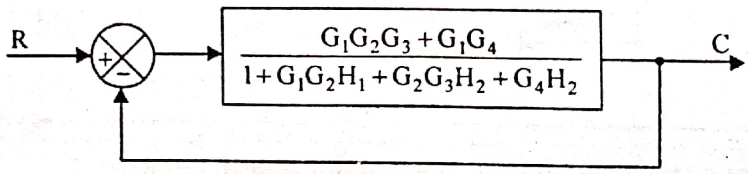


Step 6: Eliminating the feedback path



$$\frac{\dfrac{G_1(G_2G_3+G_4)}{1+G_1G_2H_1}}{1+\dfrac{G_1(G_2G_3+G_4)}{1+G_1G_2H_1}\,\dfrac{H_2}{G_1}} \Rightarrow \frac{\dfrac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1}}{\dfrac{1+G_1G_2H_1+G_2G_3H_2+G_4H_2}{1+G_1G_2H_1}} \Rightarrow \frac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2}$$

Step 7: Eliminating the feedback path



$$\frac{C}{R}=\frac{\dfrac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2}}{1+\dfrac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2}}=\frac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2+G_1G_2G_3+G_1G_4}$$

RESULT

The overall transfer function is given by,

$$\frac{C}{R}=\frac{G_1G_2G_3+G_1G_4}{1+G_1G_2H_1+G_2G_3H_2+G_4H_2+G_1G_2G_3+G_1G_4}$$

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EXAMPLE 1.18

Determine the overall transfer function $\frac{C(s)}{R(s)}$ for the system shown in fig 1.

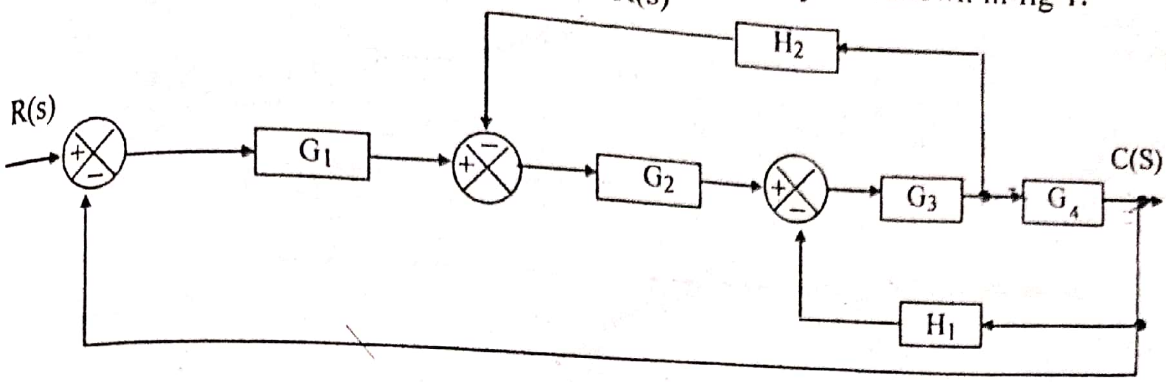
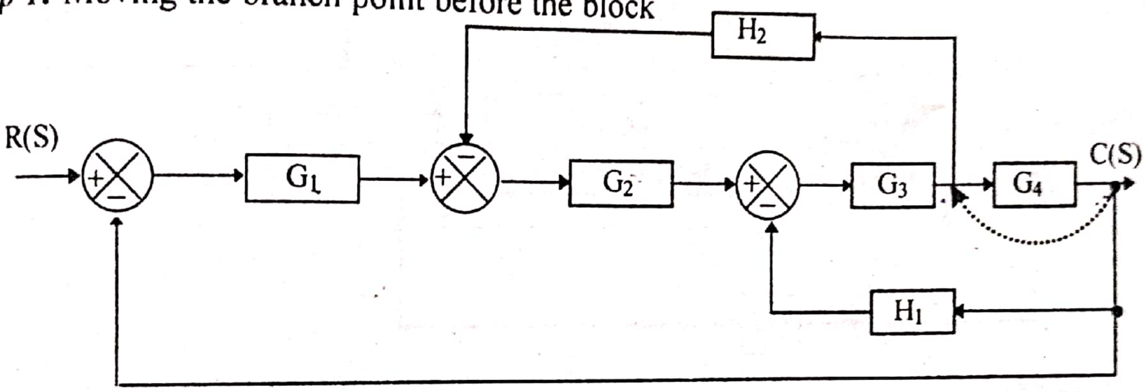


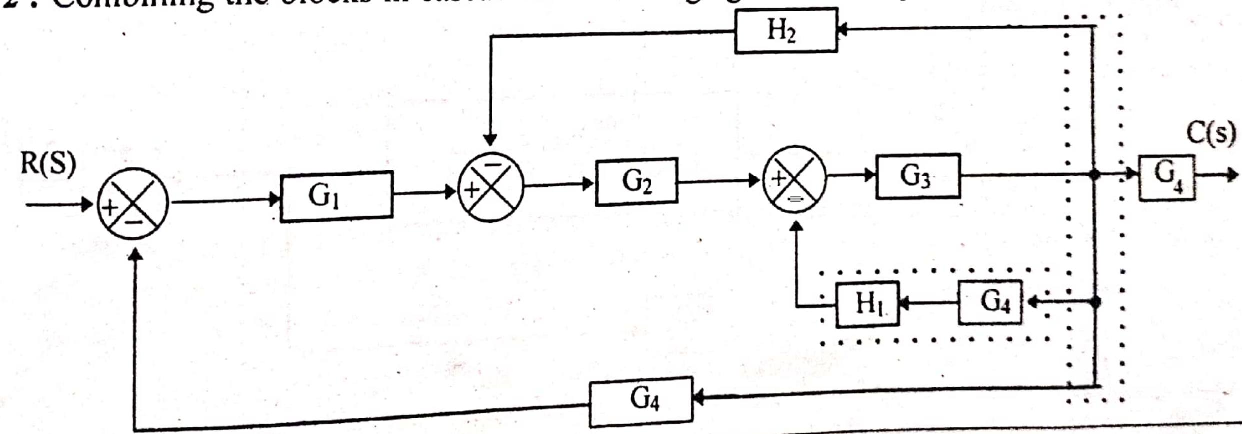
Fig 1

SOLUTION

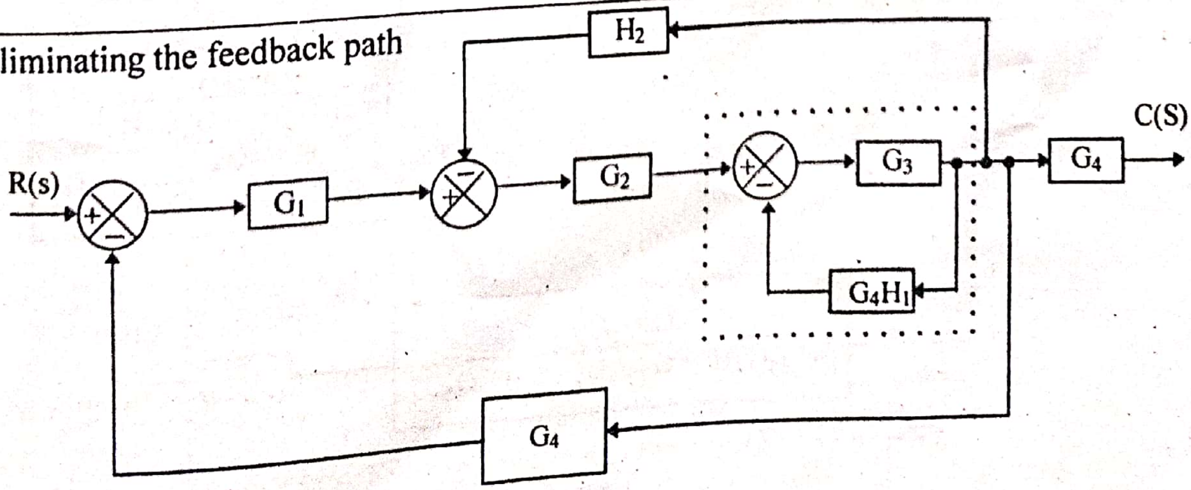
Step 1: Moving the branch point before the block



Step 2 : Combining the blocks in cascade and rearranging the branch points



Step 3 : Eliminating the feedback path



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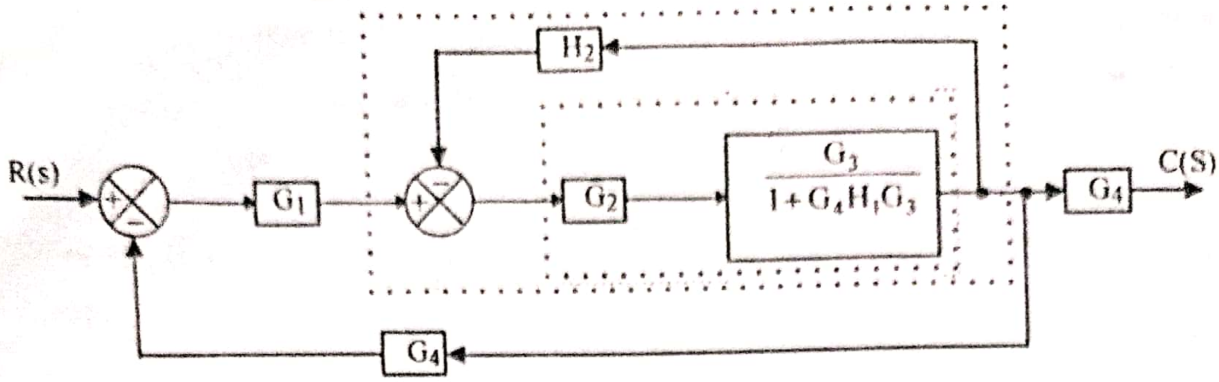
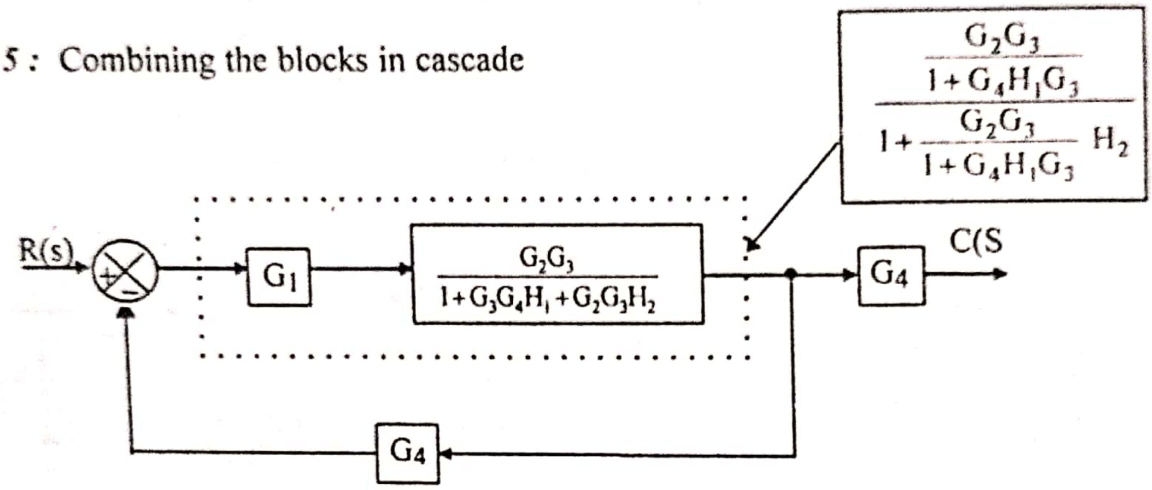
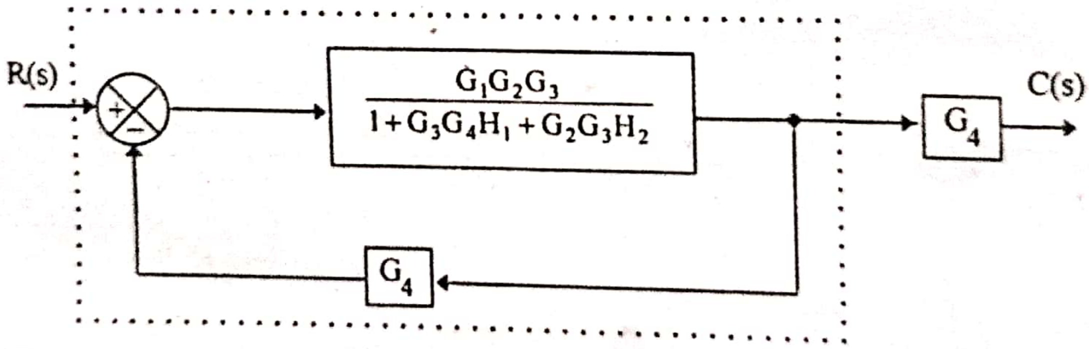
Step 4 : Combining the blocks in cascade and eliminating feedback path



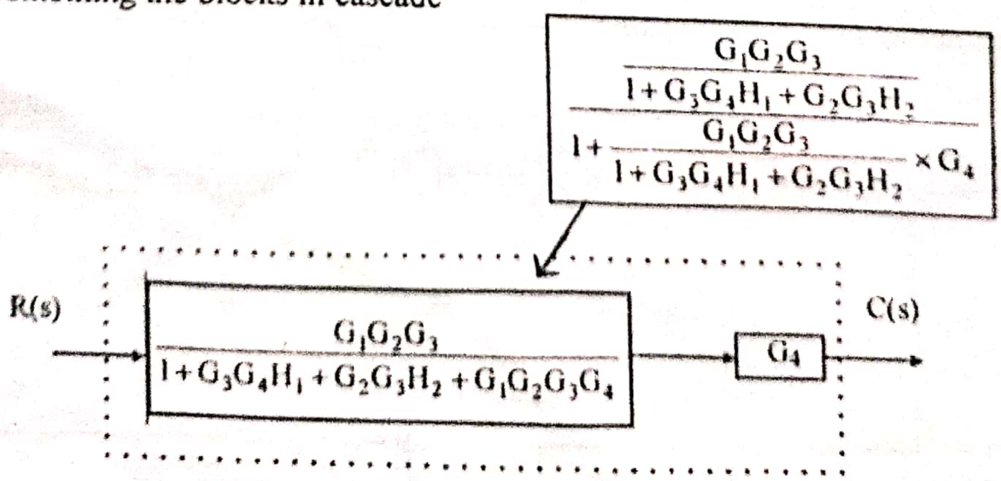
Step 5 : Combining the blocks in cascade



Step 6 : Eliminating the feedback path



Step 7 : Combining the blocks in cascade



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