Example 6.32 Design a high pass filter using window, with a cut-off frequency of 1.2 radians/sec and N = 9.

AU 2006

Solution Given

$$\Omega_c = 1.2 \text{ radians/sec}$$

If $T = 1$ sec

$$\omega_c = \Omega_c T = 1.2 \text{ radians}$$

The impulse response of a highpass filter with a cut off frequency ω_c is

$$h_d(n) = \frac{-\sin \omega_c n}{\pi} \quad |n| > 0$$

$$= 1 - \frac{\omega_c}{\pi} \quad \text{for } n = 0$$

$$\omega_c = 1.2$$

$$h_d(0) = 1 - \frac{1.2}{\pi} = 0.618$$

$$h_d(-1) = h_d(1) = \frac{-\sin 1.2}{\pi} = -0.2966$$

$$h_d(-2) = h_d(2) = \frac{-sin2.4}{2\pi} = -0.1075$$

$$h_d(-3) = h_d(3) = \frac{-sin3.6}{3\pi} = 0.0469$$

$$h_d(-4) = h_d(4) = \frac{-sin4.8}{4\pi} = 0.0719$$

Hamming window for $-4 \leq n \leq 4$ is

$$w_H(n) = 0.54 + 0.46 \cos \frac{2\pi n}{8} \quad \text{for } -4 \leq n \leq 4$$

$$= 0.54 + 0.46 \cos \frac{\pi n}{4}$$

$$w_H(0) = 1$$

$$w_H(-1) = w_H(1) = 0.865$$

$$w_H(-2) = w_H(2) = 0.54$$

$$w_H(-3) = w_H(3) = 0.215$$

$$w_H(-4) = w_H(4) = 0.08$$

$$h(n) = h_d(n) w_H(n)$$

$$h(0) = (0.618)(1) = 0.618$$

$$h(-1) = h(1) = (-0.2966)(0.865) = -0.256$$

$$h(-2) = h(2) = -0.058$$

$$h(-3) = h(3) = 0.01$$

$$h(-4) = h(4) = 0.0057$$

The causal filter coefficients are

$$h(0) = h(8) = 0.0057$$

$$h(1) = h(7) = 0.01$$

$$h(2) = h(6) = -0.058$$

Example 6.33 A bandpass FIR filter of length 7 is required. It is to have lower and upper cut-off frequencies of 3 kHz respectively and is intended to be used with a sampling frequency of 24 kHz. Determine the filter coefficients using Hanning window. Consider the filter to be causal. AU 2007

Solution

Given $N = 7$

$$f = 24\ kHz$$

$$f_{c1} = 3\ kHz,\ \ f_{c2} = 6\ kHz$$

$$\omega_{c1} = \frac{2\pi(3000)}{24000} = \frac{\pi}{4}$$

$$\omega_{c2} = \frac{2\pi(6000)}{24000} = \frac{\pi}{2}$$

$$h_d(0) = \frac{\omega_{c2} - \omega_{c1}}{\pi}$$

$$h_d(n) = \frac{1}{n\pi}\left[\sin\omega_{c2}n - \sin(\omega_{c1}n)\right];\ \ |n| > 0$$

For $N = 7$

$$h_d(0) = \frac{\frac{\pi}{2} - \frac{\pi}{4}}{\pi} = 0.25$$

$$h_d(-1) = h_d(1) = \frac{1}{\pi}\left[\sin\frac{\pi}{2} - \sin\frac{\pi}{4}\right] = 0.093$$

$$h_d(-2) = h_d(2) = \frac{1}{2\pi}\left[\sin\pi - \sin\frac{\pi}{2}\right] = -0.159$$

$$h_d(-3) = h_d(3) = \frac{1}{3\pi}\left[\sin\frac{3\pi}{2} - \sin\frac{3\pi}{4}\right] = -0.181$$

For $N = 7$

The Hamming window sequence is

$$w_H(n) = 0.54 + 0.46\cos\frac{2\pi n}{6} \quad \text{for} \quad -3 \le n \le 3$$

$$w_H(0) = 1$$

$$w_H(-1) = w_H(1) = 0.77$$

$$w_H(-2) = w_H(2) = 0.31$$

$$w_H(-3) = w_H(3) = 0.08$$

$$h(n) = h_d(n)w_H(n) \quad \text{for} \quad -3 \le n \le 3$$

$$h(0) = 0.25$$

$$h(-1) = h(1) = 0.093(0.77) = 0.07161$$

$$h(-2) = h(2) = -0.159(0.31) = -0.049$$

$$h(-3) = h(3) = -0.181(0.08) = -0.0145$$

Example 6.37 Design a bandpass and pass filter to pass frequencies in the range 1-2rad/sec using Hamming window N = 5. May'05 (set 1)

Solution

$$\omega_{c1} = 1 rad|sec \quad \omega_{c2} = 2 radians|\sec$$

If T = 1 rad/sec, we get the cut off frequencies as radian and 2 radians

$$H_d\left(e^{j\omega}\right) = 1 \text{ for } 1 \leq |\omega| \leq 2$$
$$= 0 \text{ for } 0 \leq |\omega| \leq 1 \text{ and } 2 \leq |\omega| \leq \pi$$

From example 6.7 we have

$$h_d(n) = \frac{1}{\pi n}\left[\sin 2n - \sin n\right]$$

$$h_d(0) = \frac{1}{\pi} = 0.3183$$

$$h_d(-1) = h_d(1) = \frac{\sin 2 - \sin 1}{\pi} = 0.0216$$

$$h_d(-2) = h_d(2) = \frac{\sin 4 - \sin 2}{2\pi} = -0.265$$

Hamming window for N = 5

$$\omega_{Hn}(n) = 0.5 + 0.5\cos\frac{2\pi n}{N-1} \text{ for } \frac{-(N-1)}{2} \leq n \leq \frac{(N-1)}{2}$$

$$\omega_{Hn}(n) = 0.5 + 0.5\cos\frac{\pi n}{2} \text{ for } -2 \leq n \leq 2$$

$$\omega_{Hn}(0) = 1$$

$$\omega_{Hn}(-1) = \omega_{Hn}(1) = 0.5$$

$$\omega_{Hn}(-2) = \omega_{Hn}(2) = 0$$

The filter coefficients are

$$h(0) = 0.3183$$

$$h(-1) = h(1) = 0.0108$$

Example 6.38 Design a low pass filter using rectangular window by taking ... of w(n) and with cut-off frequency of 1.2 rad/sec

Nov'05

Solution

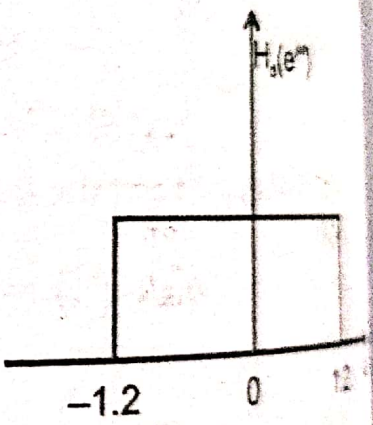
$$\omega_C = 1.2 \text{ rad/sec}$$

$$\text{If } \quad T = 1 \text{ sec}$$

The cutoff frequency $\omega_C = 1.2$ rad

$$H_d\left(e^{j\omega}\right) = 1 \text{ for } |\omega| < 1.2$$

$$= 0 \text{ for } 1.2 \leq |\omega| \leq \pi$$



$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d\left(e^{j\omega}\right) e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi} \int_{-1.2}^{1.2} e^{j\omega n} d\omega$$

$$= \frac{\sin(1.2n)}{\pi n}$$

For N = 7

$$h_d(0) = \frac{1.2}{\pi} = 0.387$$

$$h_d(-1) = h_d(1) = 0.2967$$

$$h_d(-2) = h_d(2) = 0.1075$$

$$h_d(-3) = h_d(3) = -0.0469$$

ample 6.41 Design a high pass filter using hamming window with a cutoff frequency of 1.2 radians/sec and N = 9. Nov'05

ution

Given $\omega_c = 1.2$ rad/sec

The coefficients of high pass filter

$$h_d(0) = 1 - \frac{\omega_C}{\pi} = 0.618$$

$$h_d(n) = \frac{-\sin \omega_C}{\pi n} \quad |n| > 0$$

$$h_d(-1) = h_d(1) = \frac{-\sin 1.2}{\pi} = -0.2966$$

$$h_d(-2) = h_d(2) = \frac{-\sin 2.4}{2\pi} = -0.1075$$

$$h_d(-3) = h_d(3) = \frac{-\sin 3.6}{3\pi} = -0.04695$$

$$h_d(-4) = h_d(4) = \frac{-\sin 4.8}{4\pi} = 0.07927$$

ming window

$$\omega_H(n) = 0.54 + .46 \cos \frac{2\pi n}{N-1} \text{ for } |n| \leqslant \frac{N-1}{2}$$

$$\omega_H(-1) = \omega_H(1) = 0.54 + 0.46 \cos \frac{2\pi}{8} = 0.865$$

$$\omega_H(-2) = \omega_H(2) = 0.54 + 0.46 \cos \frac{\pi}{2} = 0.54$$

$$\omega_H(-3) = \omega_H(3) = 0.54 + 0.46 \cos \frac{3\pi}{4} = 0.2147$$

$$\omega_H(-4) = \omega_H(4) = 0.54 + 0.46 \cos \pi = 0.08$$

$$\omega_H(0) = 1$$

The causal filter coefficients are

$$
\begin{aligned}
h_d(4) &= 0.618 \\
h_d(3) &= h_d(5) = -0.256 \\
h_d(2) &= h_d(6) = -0.058 \\
h_d(1) &= h_d(7) = 0.01 \\
h_d(0) &= h_d(8) = 0.00634
\end{aligned}
$$