

Example 6.9 Repeat the example 6.6 using (a) Hanning window (b) Hamming window

Solution

$$H_d(e^{j\omega}) = 1 \quad \text{for } \frac{\pi}{4} \leq |\omega| \leq \pi$$

$$= 0 \quad \text{for } |\omega| \leq \frac{\pi}{4}$$

(a) Hanning window

$$w_{Hn}(n) = 0.5 + 0.5 \cos \frac{2\pi n}{N-1} \quad \text{for } -(N-1)/2 \leq n \leq (N-1)/2$$

$$= 0 \quad \text{otherwise}$$

For $N = 11$

$$w_{Hn}(n) = 0.5 + 0.5 \cos \frac{\pi n}{5} \quad -5 \leq n \leq 5$$

$$= 0 \quad \text{otherwise}$$

$$w_{Hn}(0) = 0.5 + 0.5 = 1$$

$$w_{Hn}(1) = w_{Hn}(-1) = 0.5 + 0.5 \cos \frac{\pi}{5} = 0.9045$$

$$w_{Hn}(2) = w_{Hn}(-2) = 0.5 + 0.5 \cos \frac{2\pi}{5} = 0.655$$

$$w_{Hn}(3) = w_{Hn}(-3) = 0.5 + 0.5 \cos \frac{3\pi}{5} = 0.345$$

$$w_{Hn}(4) = w_{Hn}(-4) = 0.5 + 0.5 \cos \frac{4\pi}{5} = 0.0945$$

$$w_{Hn}(5) = w_{Hn}(-5) = 0.5 + 0.5 \cos \pi = 0$$

The filter coefficients can be obtained from the example 6.6, i.e.,

$$h_d(n) = \frac{\sin \pi n - \sin \frac{\pi}{4} n}{n\pi}$$

$$h_d(0) = \lim_{n \rightarrow 0} \left[\frac{\sin \pi n - \sin \frac{\pi}{4} n}{\pi n} \right] = 1 - \frac{1}{4} = 0.75$$

$$h_d(-1) = h_d(1) = \frac{\sin \pi - \sin \frac{\pi}{4}}{\pi} = -0.225$$

$$h_d(-2) = h_d(2) = \frac{\sin 2\pi - \sin \frac{\pi}{2}}{2\pi} = -0.159$$

$$h_d(-3) = h_d(3) = \frac{\sin 3\pi - \sin \frac{3\pi}{4}}{3\pi} = -0.075$$

$$h_d(-4) = h_d(4) = \frac{\sin 4\pi - \sin \pi}{4\pi} = 0$$

$$h_d(-5) = h_d(5) = \frac{\sin 5\pi - \sin \frac{5\pi}{4}}{5\pi} = 0.045$$

The filter coefficients using Hanning window are

$$h(n) = h_d(n)w_{Hn}(n) \quad \text{for } -5 \leq n \leq 5$$

$$= 0 \quad \text{otherwise}$$

$$h(0) = h_d(0)w_{Hn}(0) = (0.75)(1) = 0.75$$

$$h(-1) = h(1) = h_d(1)w_{Hn}(1) = (-0.225)(0.905) = -0.204$$

$$h(-2) = h(2) = h_d(2)w_{Hn}(2) = (-0.159)(0.655) = -0.104$$

$$h(-3) = h(3) = h_d(3)w_{Hn}(3) = (-0.075)(0.345) = -0.026$$

$$h(-4) = h(4) = h_d(4)w_{Hn}(4) = (0)(0.8145) = 0$$

$$h(-5) = h(5) = h_d(5)w_{Hn}(5) = (0.045)(0) = 0$$

The transfer function of the filter is given by

$$H(z) = h(0) + \sum_{n=1}^5 h(n) [z^{-n} + z^n]$$

$$= 0.75 - 0.204(z + z^{-1}) - 0.104(z^2 + z^{-2}) - 0.026(z^3 + z^{-3})$$

The transfer function of the realizable filter is

$$H'(z) = z^{-5}H(z)$$

$$= -0.026z^{-2} - 0.104z^{-3} - 0.204z^{-4} + 0.75z^{-5} - 0.204z^{-6}$$

$$- 0.104z^{-7} - 0.026z^{-8}$$