

Example 4.6 Find the DFT of a sequence $x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$ using DIT algorithm.

Solution

The twiddle factors associated with the flow graph are

$$W_8^0 = 1; W_8^1 = (e^{-j2\pi/8})^1 = e^{-j\pi/4} = 0.707 - j0.707$$

$$W_8^2 = (e^{-j2\pi/8})^2 = e^{-j\pi/2} = -j$$

$$W_8^3 = (e^{-j2\pi/8})^3 = e^{-j3\pi/4} = -0.707 - j0.707$$

The basic operation is

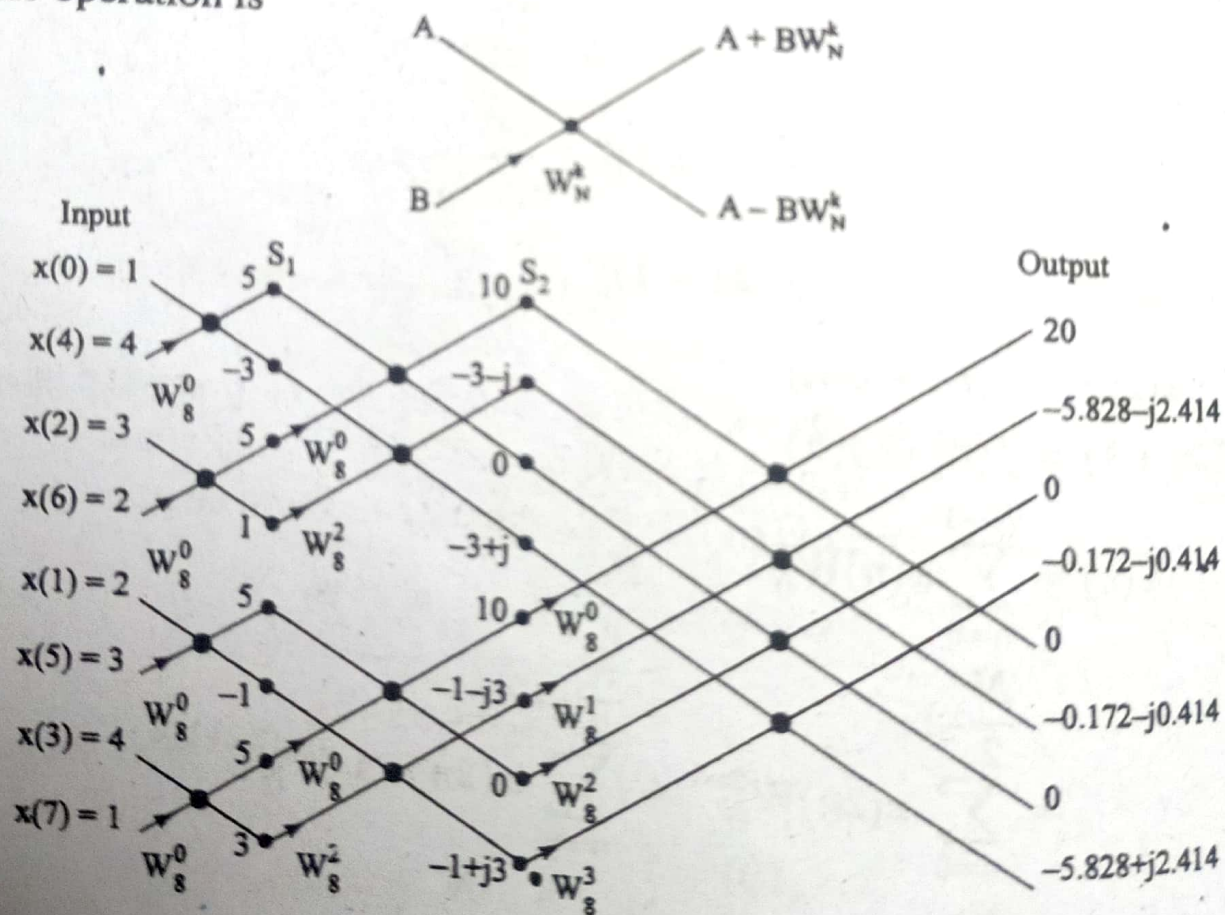


Fig. 4.8(a)

(Or)

Fig. 4.16

Ans: First stage

Example 4.8 Repeat the example 4.6 using DIF algorithm.

Solution

The twiddle factors associated with the flow graph are $W_8^0 = 1$, $W_8^1 = 0.707 - j0.707$, $W_8^2 = -j$, $W_8^3 = -0.707 - j0.707$. The basic operation is

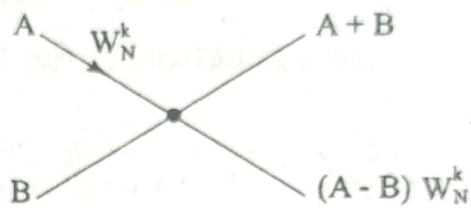


Fig. 4.17

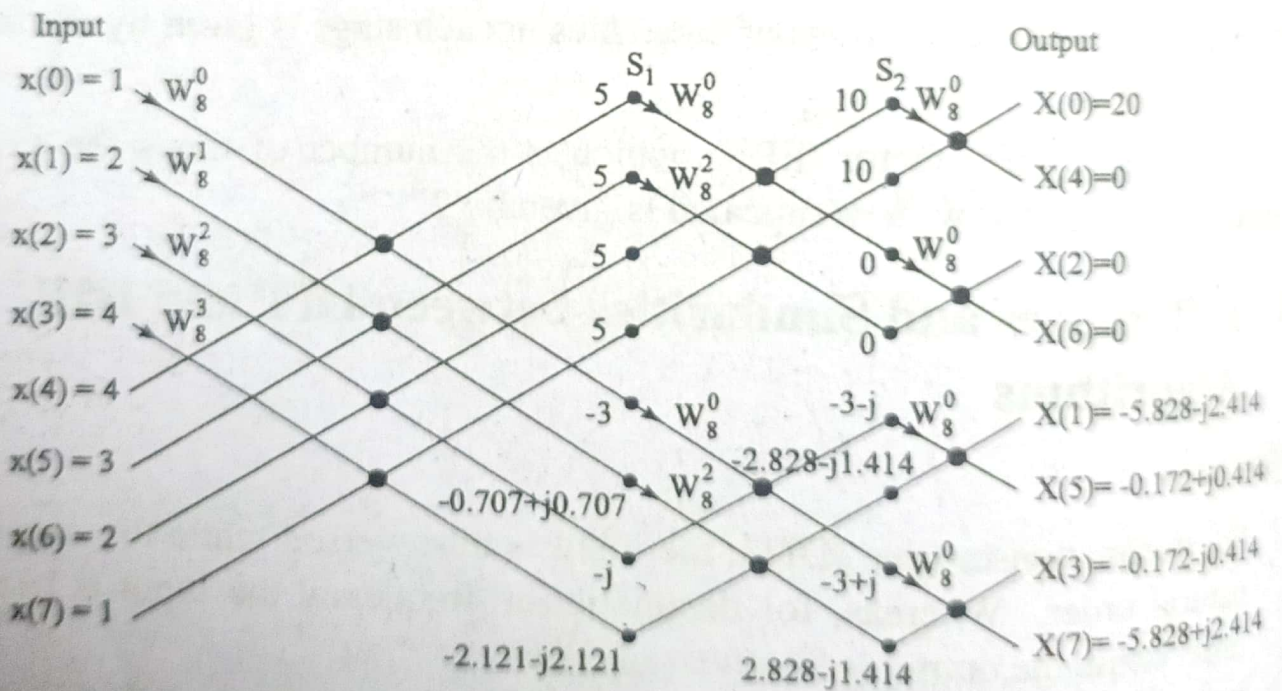


Fig. 4.18

Example 4.9 Compute the eight-point DFT of the sequence

$$x(n) = \begin{cases} 1 & 0 \leq n \leq 7 \\ 0 & \text{otherwise} \end{cases}$$

EIE AU'03

CSE AU'03

by using DIT, DIF algorithms.

Solution

DIT algorithm

The twiddle factors associated with butterflies can be found as

$$W_8^0 = 1; W_8^1 = 0.707 - j0.707; W_8^2 = -j; W_8^3 = -0.707 - j0.707$$

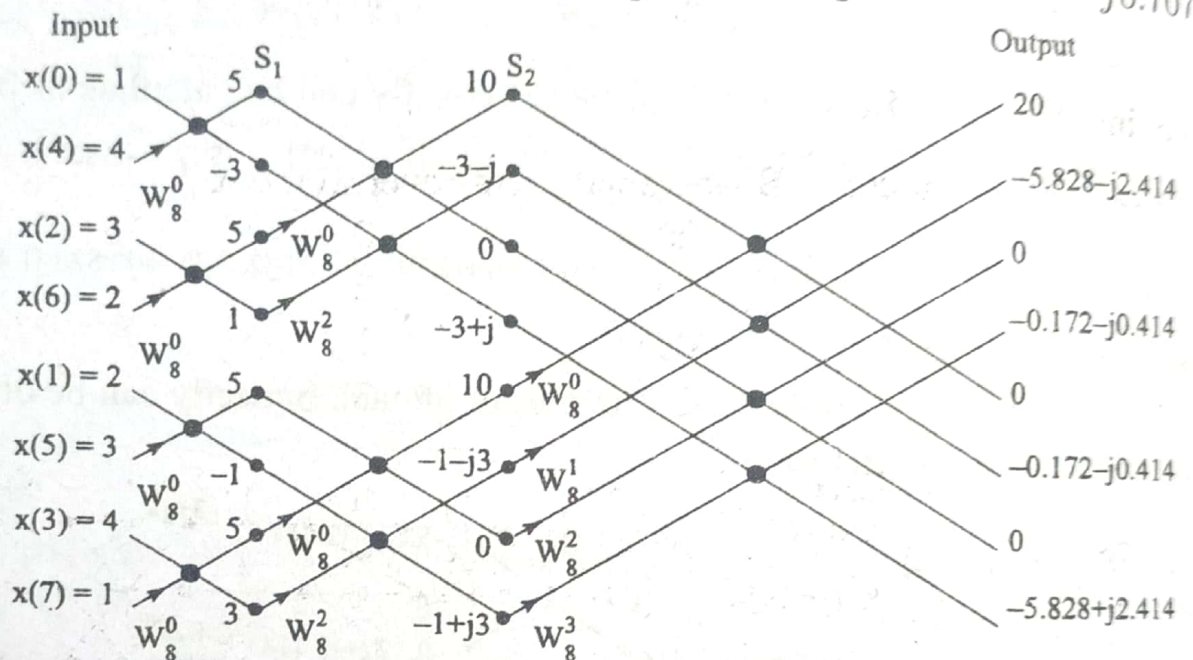


Fig. 4.19

$$X(k) = \{8, 0, 0, 0, 0, 0, 0, 0\}$$

DIF algorithm

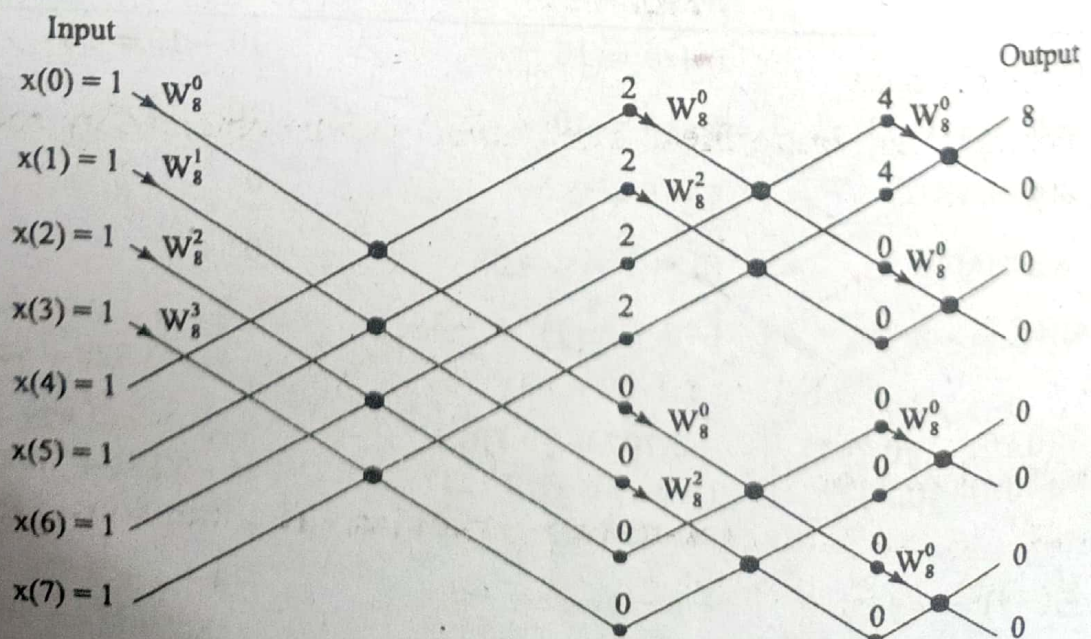


Fig. 4.20

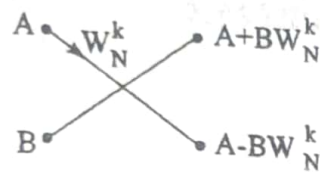
$$X(k) = \{8, 0, 0, 0, 0, 0, 0, 0\}$$

Example 4.10 Compute 4-point DFT of a sequence $x(n) = \{0, 1, 2, 3\}$ using DIT, DIF algorithm.

Solution

DIT algorithm

The butterfly operation is given by



Twiddle factors associated with butterflies are

$$W_4^0 = 1; \quad W_4^1 = e^{-j2\pi/4} = -j$$

Bit reversal of input is given by

Input index	Binary index	Bit-reversal	Bit-reversal index
0	00	00	0
1	01	10	2
2	10	01	1
3	11	11	3

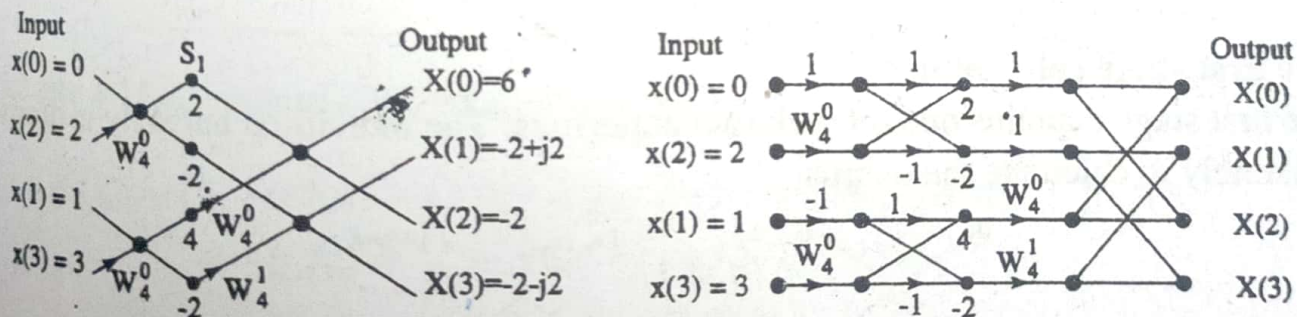


Fig. 4.21

4.11 Compute IDFT of the sequence

$$X(k) = \{7, -0.707 - j0.707, -j, 0.707 - j0.707, 1, 0.707 + j0.707, j, -0.707 + j0.707\}$$

(EIE AU' 03)

DIT algorithm.

Take complex conjugate of $X(k)$ and apply bit reversal index inputs to flow graph

8-point DIT algorithm

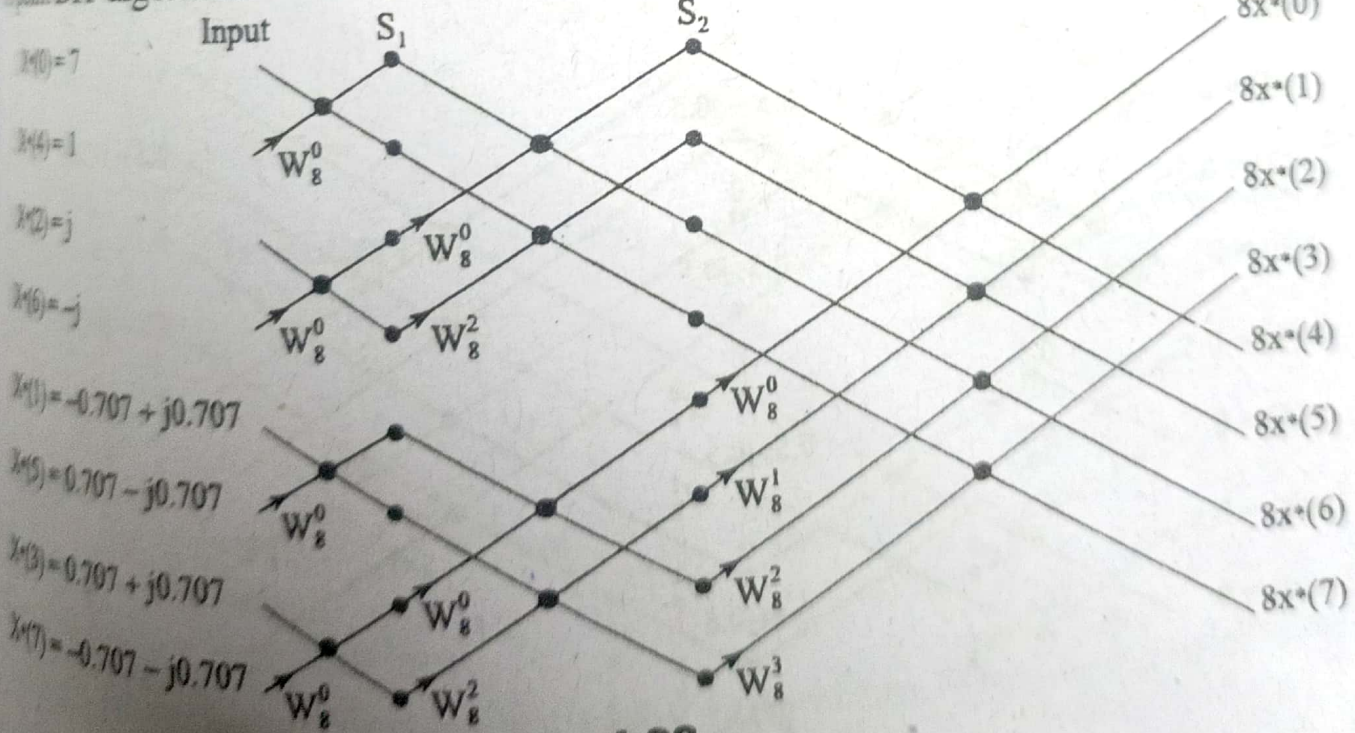


Fig. 4.23

4.34 Digital Signal Processing

Input	S_1	S_2	Output
7	$7 + 1 = 8$	$8 + 0 = 8$	$8 + 0 = 8$
1	$7 - 1 = 6$	$6 + (-j)2j = 8$	$8 + 0(W^1) = 8$
j	$j - j = 0$	$8 - 0 = 8$	$8 + 0(W^2) = 8$
$-j$	$j - (-j) = 2j$	$6 - (-j)(2j) = 4$	$4 + (-0.707 - j0.707)(-2.828 + j2.828) = 8$
$-0.707 + j0.707$	$(-0.707 + j0.707) + (0.707 - j0.707) = 0$	0	$8 - 0 = 8$
$0.707 - j0.707$	$(-0.707 + j0.707) - (0.707 - j0.707) = -1.414 + j1.414$	$(-1.414 + j1.414) + (-j)(1.414 + j1.414) = 0$	$8 - 0(W^1) = 8$
$0.707 + j0.707$	$(0.707 + j0.707) + (-0.707 - j0.707) = 0$	0	$8 - 0(W^2) = 8$
$-0.707 - j0.707$	$0.707 + j0.707 - (-0.707 - j0.707) = 1.414 + j1.414$	$(-1.414 + j1.414) - (-j)(1.414 + j1.414) = -2.828 + j2.828$	$4 - (-0.707 - j0.707)(-2.828 + j2.828) = 0$

Output

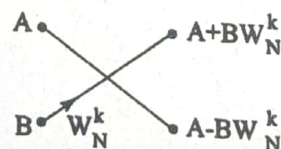
$Nx^*(n) = \{8, 8, 8, 8, 8, 8, 8, 0\}$

$x(n) = \{1, 1, 1, 1, 1, 1, 1, 0\}$

Example 4.12 Compute the eight-point DFT of the sequence $x(n) = \{0.5, 0.5, 0.5, 0.5, 0, 0, 0, 0\}$ using the inplace radix-2 DIT algorithm.

Solution

The basic butterfly operation is



The twiddle factors are

$$W_8^0 = 1; W_8^1 = 0.707 - j0.707; W_8^2 = -j; W_8^3 = -0.707 - j0.707$$

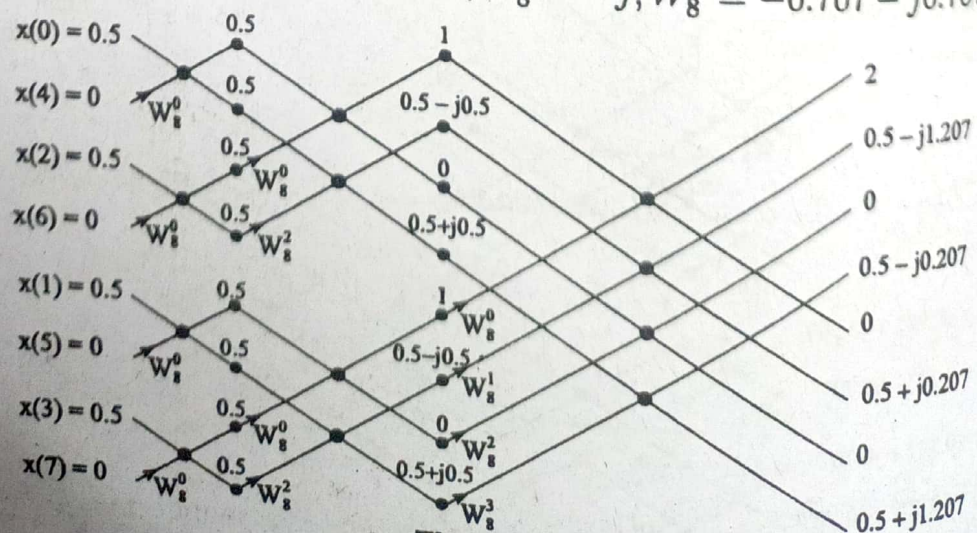


Fig. 4.24

The Fast Fourier Transform 4.35

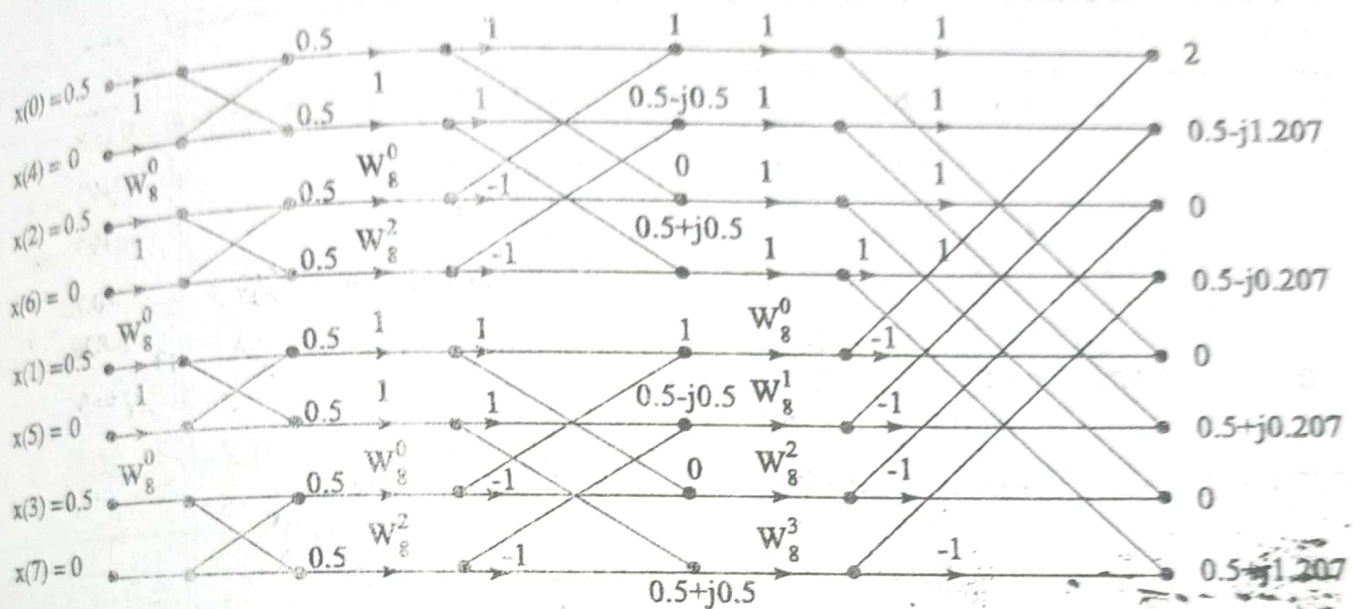


Fig. 4.25

Apply the input to the butterfly of DFT algorithm in bit-reversed order and

Example 4.14 Compute the DFT of a sequence $x(n) = \{1, -1, 1, -1\}$ using DIT algorithm.

Solution

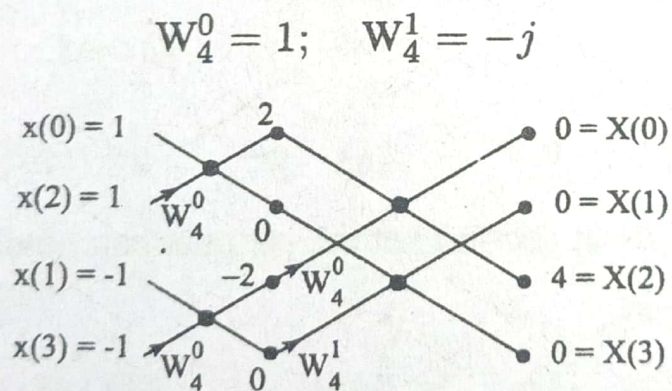


Fig. 4.27

$$X(k) = \{0, 0, 4, 0\}$$

Example 4.15 Find the DFT of the sequence $x(n) = \{1, 0, 0, 1\}$ using DIF algorithm.

Solution

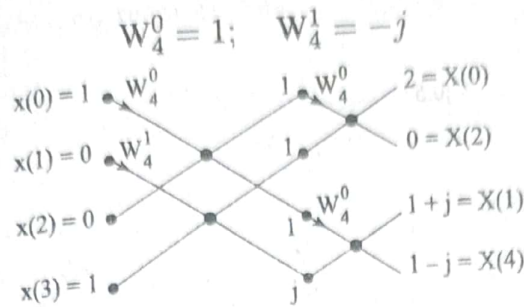


Fig. 4.28

$$X(k) = \{2, 1+j, 0, 1-j\}$$

Example 4.16 Evaluate and compare the 8-point for the following sequences using DIT-FFT algorithm.

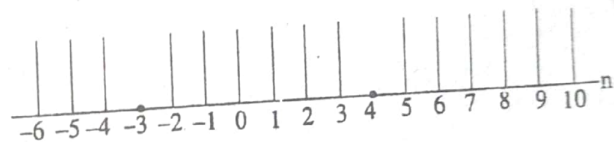
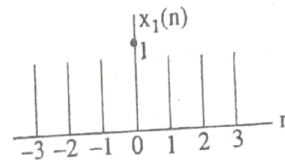
a. $x_1(n) = \begin{cases} 1 & \text{for } -3 \leq n \leq 3 \\ 0 & \text{otherwise} \end{cases}$ b. $x_2(n) = \begin{cases} 1 & \text{for } 0 \leq n \leq 6 \\ 0 & \text{otherwise} \end{cases}$

Solution

Given $x_1(n) = \begin{cases} 1 & \text{for } -3 \leq n \leq 3 \\ 0 & \text{otherwise} \end{cases}$

The sequence $x_1(n)$ for $N=8$ is

$$x_1(n) = \{1, 1, 1, 1, 0, 1, 1, 1\}$$



Twiddle factors are given by

$$W^0 = 1; W^1 = 0.707 - j0.707; W^2 = -j; W^3 = -0.707 - j0.707$$

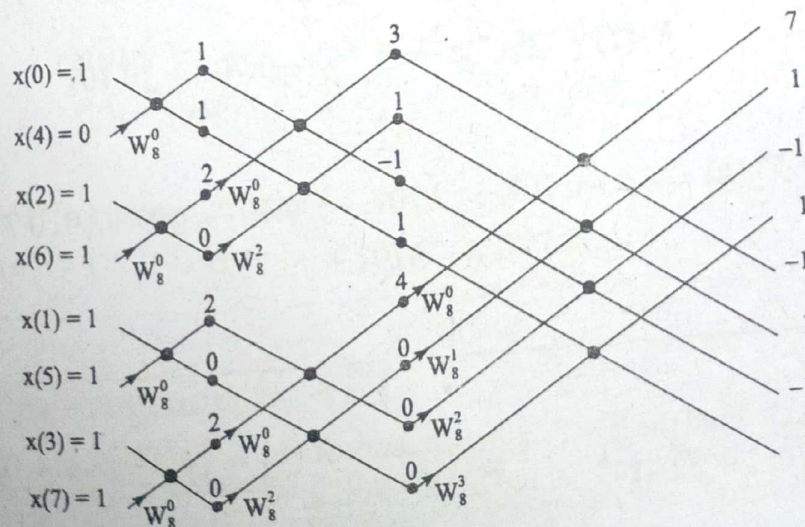


Fig. 4.29

4.38 Digital Signal Processing

$$X_1(k) = \{7, 1, -1, 1, -1, 1, -1, 1\}$$

$$(b) x_2(n) = \{1, 1, 1, 1, 1, 1, 1, 0\}$$

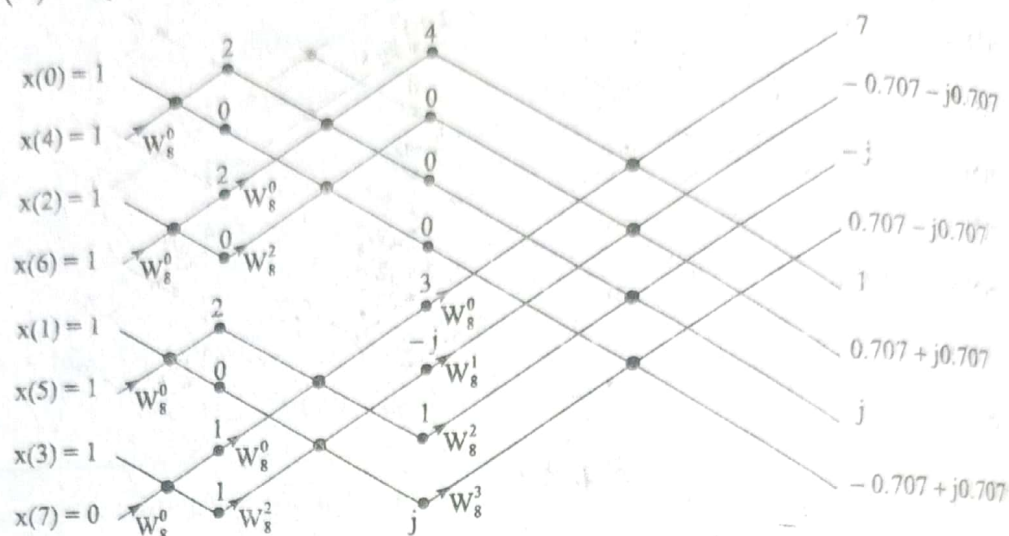


Fig. 4.30

The sequence $x_2(n)$ can be obtained by shifting the sequence $x_1(n)$ right three times. Therefore using time shifting property of DFT we have the relation

$$X_2(k) = e^{-j6\pi k/8} X_1(k)$$

$$= e^{-j3\pi k/4} X_1(k)$$

$$X_2(0) = X_1(0)e^0 = 7$$

$$X_2(1) = X_1(1)e^{-j3\pi/4} = -0.707 - j0.707$$

$$X_2(2) = X_1(2)e^{-j3\pi/2} = -j$$

$$X_2(3) = X_1(3)e^{-j9\pi/4} = 0.707 - j0.707$$

$$X_2(4) = X_1(4)e^{-j3\pi} = 1$$

$$X_2(5) = X_1(5)e^{-j15\pi/4} = 0.707 + j0.707$$

$$X_2(6) = X_1(6)e^{-j9\pi/2} = j$$

$$X_2(7) = X_1(7)e^{-j21\pi/4} = -0.707 + j0.707$$

$$X_2(k) = \{7, -0.707 - j0.707, -j, 0.707 - j0.707, 1, 0.707 + j0.707, j, -0.707 + j0.707\}$$

Example 4.17 Find the IDFT of the sequence $X(k) = \{4, 1 - j2.414, 0, 1 - j0.414, 0, 1 + j0.414, 0, 1 + j2.414\}$ using DIF algorithm.

Solution

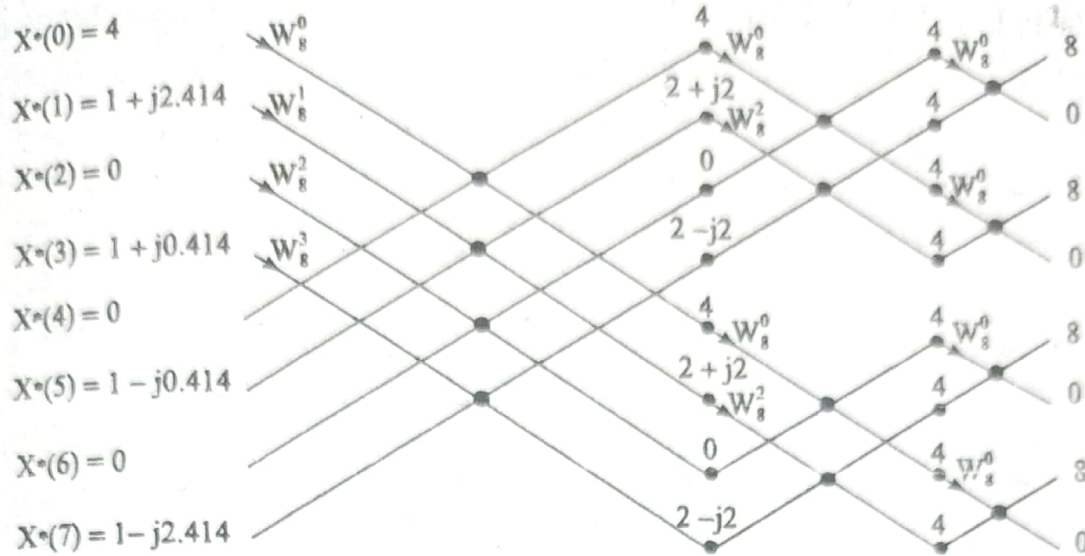


Fig. 4.31

The output $8x^*(n)$ is in bit reversal order.
Therefore

$$x(n) = \{1, 1, 1, 1, 0, 0, 0, 0\}$$

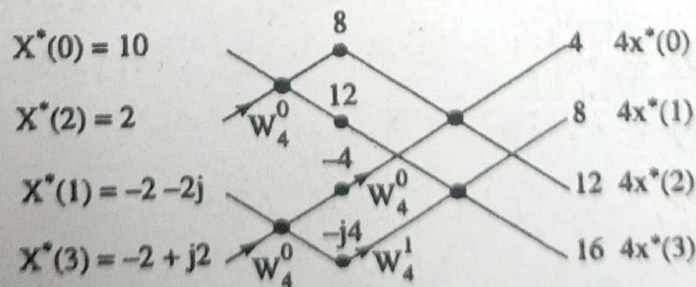
Example 4.18 Find the IDFT of the sequence

$$X(k) = \{10, -2 + j2, -2, -2 - j2\}$$

using DIT algorithm.

Solution

Then twiddle factors are $W_4^0 = 1$; $W_4^1 = -j$



The output $Nx^*(n)$ is normal order.
Therefore $x(n) = \{1, 2, 3, 4\}$.