Window type	Window sequence $w(n)$ **for** $-(N-1)/2 \le n \le (N-1)/2$
Rectangular	1
Triangular window	$1 - \frac{2\|n\|}{N-1}$
Raised cosine window	$\alpha + (1-\alpha)\cos\frac{2\pi n}{N-1}$ $\alpha = 0.5$ for Hanning window $\alpha = 0.54$ for Hamming window

Window type	Window sequence $w(b)$ for $-(N-1)/2 \le n \le (N-1)/2$	
Blackman window	$0.42 + 0.5\cos\frac{2\pi n}{N-1} + 0.08\cos\frac{4\pi n}{N-1}$	0. + +
Kaiser window	$\omega_k(n) = \frac{I_0\left[\alpha\sqrt{1-\left(\frac{2n}{N-1}\right)^2}\right]}{I_0(\alpha)}$	

Example 6.9 Repeat the example 6.6 using (a) Hanning window (b) Hamming window

$H_d(e^{j\omega}) = 1$ for $\pi/4 \le |\omega| \le \pi$
$= 0$ for $|\omega| \le \pi/4$

Solution

(a) Hanning window

$$w_{Hn}(n) = 0.5 + 0.5\cos\frac{2\pi n}{N-1} \quad \text{for } -(N-1)/2 \le n \le (N-1)/2$$
$$= 0 \quad \text{otherwise}$$

For $N = 11$

$$w_{Hn}(n) = 0.5 + 0.5\cos\frac{\pi n}{5} \quad -5 \le n \le 5$$
$$= 0 \quad \text{otherwise}$$

$$w_{Hn}(0) = 0.5 + 0.5 = 1$$
$$w_{Hn}(1) = w_{Hn}(-1) = 0.5 + 0.5\cos\frac{\pi}{5} = 0.9045$$
$$w_{Hn}(2) = w_{Hn}(-2) = 0.5 + 0.5\cos\frac{2\pi}{5} = 0.655$$
$$w_{Hn}(3) = w_{Hn}(-3) = 0.5 + 0.5\cos\frac{3\pi}{5} = 0.345$$
$$w_{Hn}(4) = w_{Hn}(-4) = 0.5 + 0.5\cos\frac{4\pi}{5} = 0.0945$$
$$w_{Hn}(5) = w_{Hn}(-5) = 0.5 + 0.5\cos\pi = 0$$

The filter coefficients can be obtained from the example 6.6, i.e.,

$$h_d(n) = \frac{\sin\pi n - \sin\frac{\pi}{4}n}{n\pi}$$
$$h_d(0) = \lim_{n\to 0}\left[\frac{\sin\pi n - \sin\frac{\pi}{4}n}{\pi n}\right] = 1 - \frac{1}{4} = 0.75$$
$$h_d(-1) = h_d(1) = \frac{\sin\pi - \sin\frac{\pi}{4}}{\pi} = -0.225$$
$$h_d(-2) = h_d(2) = \frac{\sin 2\pi - \sin\frac{\pi}{2}}{2\pi} = -0.159$$
$$h_d(-3) = h_d(3) = \frac{\sin 3\pi - \sin\frac{3\pi}{4}}{3\pi} = -0.075$$
$$h_d(-4) = h_d(4) = \frac{\sin 4\pi - \sin\pi}{4\pi} = 0$$
$$h_d(-5) = h_d(5) = \frac{\sin 5\pi - \sin\frac{5\pi}{4}}{5\pi} = 0.045$$

The filter coefficients using Hanning window are

$$h(n) = h_d(n)w_{Hn}(n) \quad \text{for} \quad -5 \le n \le 5$$
$$= \quad 0 \quad \text{otherwise}$$

$$h(0) = h_d(0)w_{Hn}(0) = (0.75)(1) = 0.75$$
$$h(-1) = h(1) = h_d(1)w_{Hn}(1) = (-0.225)(0.905) = -0.204$$
$$h(-2) = h(2) = h_d(2)w_{Hn}(2) = (-0.159)(0.655) = -0.104$$
$$h(-3) = h(3) = h_d(3)w_{Hn}(3) = (-0.075)(0.345) = -0.026$$
$$h(-4) = h(4) = h_d(4)w_{Hn}(4) = (0)(0.8145) = 0$$
$$h(-5) = h(5) = h_d(5)w_{Hn}(5) = (0.045)(0) = 0$$

The transfer function of the filter is given by

$$H(z) = h(0) + \sum_{n=1}^{5} h(n)\left[z^{-n} + z^{n}\right]$$
$$= 0.75 - 0.204\left(z + z^{-1}\right) - 0.104\left(z^2 + z^{-2}\right) - 0.026\left(z^3 + z^{-3}\right)$$

The transfer function of the realizable filter is

$$H'(z) = z^{-5}H(z)$$
$$-0.026z^{-2} - 0.104z^{-3} - 0.204z^{-4} + 0.75z^{-5} - 0.204z^{-6}$$
$$- 0.104z^{-7} - 0.026z^{-8}$$

(b) Hamming window

The Hamming window sequence is given by

$$\begin{aligned} w_H(n) &= 0.54 + 0.46 \cos \frac{2\pi n}{N-1} \quad \text{for} \quad -(N-1)/2 \le n \le (N-1)/2 \\ &= 0 \quad \text{otherwise} \end{aligned}$$

The window sequence for $N = 11$ is given by

$$\begin{aligned} w_H(n) &= 0.54 + 0.46 \cos \frac{\pi n}{5} \quad \text{for} \quad -5 \le n \le 5 \\ &= 0 \quad \text{otherwise} \end{aligned}$$

$$w_H(0) = 0.54 + 0.46 = 1$$

$$w_H(-1) = w_H(1) = 0.54 + 0.46\cos\frac{\pi}{5} = 0.912$$

$$w_H(-2) = w_H(2) = 0.54 + 0.46\cos\frac{2\pi}{5} = 0.682$$

$$w_H(-3) = w_H(3) = 0.54 + 0.46\cos\frac{3\pi}{5} = 0.398$$

$$w_H(-4) = w_H(4) = 0.54 + 0.46\cos\frac{4\pi}{5} = 0.1678$$

$$w_H(-5) = w_H(5) = 0.54 + 0.46\cos\pi = 0.08$$

The filter coefficients using Hamming window sequence are

$$\begin{aligned} h(n) &= h_d(n)w_H(n) \quad \text{for} \quad -5 \le n \le 5 \\ &= 0 \quad \text{otherwise} \end{aligned}$$

$$\begin{aligned} h(0) &= h_d(0)w_{Hn}(0) = (1)(0.75) = 0.75 \\ h(-1) = h(1) &= h_d(1)w_{Hn}(1) = (-0.225)(0.912) = -0.2052 \\ h(-2) = h(2) &= h_d(2)w_{Hn}(2) = (-0.159)(0.682) = -0.1084 \\ h(-3) = h(3) &= h_d(3)w_{Hn}(3) = (-0.075)(0.398) = -0.03 \\ h(-4) = h(4) &= h_d(4)w_{Hn}(4) = (0)(0.1678) = 0 \\ h(-5) = h(5) &= h_d(5)w_{Hn}(5) = (-0.045)(0.08) = 0.0036 \end{aligned}$$

The transfer function of the filter is given by

$$\begin{aligned} H(z) &= h(0) + \sum_{n=1}^{5} \left[h(n)\left(z^{-n} + z^{n}\right)\right] \\ &= 0.75 - 0.2052\left(z^{-1} + z\right) - 0.1084\left(z^{-2} + z^{2}\right) - 0.03\left(z^{-3} + z^{3}\right) \\ &\quad + 0.0036\left(z^{-5} + z^{5}\right) \end{aligned}$$

The transfer function of the realizable filter is

$$\begin{aligned} H'(z) &= z^{-5}H(z) \\ &= 0.0036 - 0.03z^{-2} - 0.1084z^{-3} - 0.2052z^{-4} + 0.75z^{-5} \\ &\quad - 0.2052z^{-6} - 0.1084z^{-7} - 0.03z^{-8} + 0.0036z^{-10} \end{aligned}$$

Example 6.10 Design a filter with

$$H_d(e^{j\omega}) = e^{-j3\omega} \qquad \frac{-\pi}{4} \le \omega \le \frac{\pi}{4}$$
$$= 0 \qquad \frac{\pi}{4} < |\omega| \le \pi$$

Using a Hamming window with $N = 7$ (AU EEE'07)

Solution

Given $H_d(e^{j\omega}) = e^{-j3\omega}$

The frequency response is having a term $e^{-j\omega(N-1)/2}$ which gives $h(n)$ symmetrical about $n = \frac{N-1}{2} = 3$, i.e., we get a causal sequence.

We have

$$h_d(n) = \frac{1}{2\pi}\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} e^{-j3\omega} e^{j\omega n} d\omega$$
$$= \frac{1}{2\pi}\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} e^{j(n-3)\omega} d\omega$$
$$= \frac{\sin\frac{\pi}{4}(n-3)}{\pi(n-3)}$$

For $N = 7$ we have

$$h_d(0) = h_d(6) = 0.075$$
$$h_d(1) = h_d(5) = 0.159$$
$$h_d(2) = h_d(4) = 0.22$$
$$h_d(3) = 0.25$$

The non-causal window sequence is

$$w_{Hn}(n) = 0.5 + 0.5\cos\frac{2\pi n}{N-1} \quad \text{for} \quad -(N-1)/2 \le n \le (N-1)/2$$
$$= 0 \quad \text{otherwise}$$

For $N = 7$

$$w_{Hn}(n) = 0.5 + 0.5\cos\frac{2\pi n}{N-1} \quad \text{for} \quad -3 \le n \le 3$$
$$= 0 \quad \text{otherwise}$$
$$w_{Hn}(0) = 0.5 + 0.5 = 1$$
$$w_{Hn}(-1) = w_{Hn}(1) = 0.5 + 0.5\cos\frac{\pi}{3} = 0.75$$

$$w_{Hn}(-2) = w_{Hn}(2) = 0.5 + 0.5\cos\frac{2\pi}{3} = 0.25$$

$$w_{Hn}(-3) = 0.5 + 0.5\cos\pi = 0$$

The causal window sequence can be obtained by shifting the sequence $w_{Hn}(n)$ to right by 3 samples, i.e.,

$$w_{Hn}(0) = w_{Hn}(6) = 0; \; w_{Hn}(1) = w_{Hn}(5) = 0.25$$

$$w_{Hn}(2) = w_{Hn}(4) = 0.75 \; \& \; w_{Hn}(3) = 1$$

The filter coefficients using Hanning window are

$$h(n) = h_d(n)w_{Hn}(n) \quad \text{for} \quad 0 \le n \le 6$$

$$h(0) = h(6) = h_d(0)w_{Hn}(0) = (0.075)(0) = 0$$

$$h(1) = h(5) = h_d(1)w_{Hn}(1) = (0.159)(0.25) = 0.03975$$

$$h(2) = h(4) = h_d(2)w_{Hn}(2) = (0.22)(0.75) = 0.165$$

$$h(3) = h_d(3)w_{Hn}(3) = (0.25)(1) = 0.25$$

Example 6.8 Design an ideal bandreject filter with a desired frequency response

$$H_d(e^{j\omega}) = 1 \quad \text{for} \quad |\omega| \le \frac{\pi}{3} \quad \text{and} \quad |\omega| \ge \frac{2\pi}{3}$$

$$= 0 \quad \text{otherwise}$$

Find the value of $h(n)$ for $N = 11$. Find $H(z)$. Plot the magnitude response.

Solution

The desired frequency response is shown in Fig. 6.14.

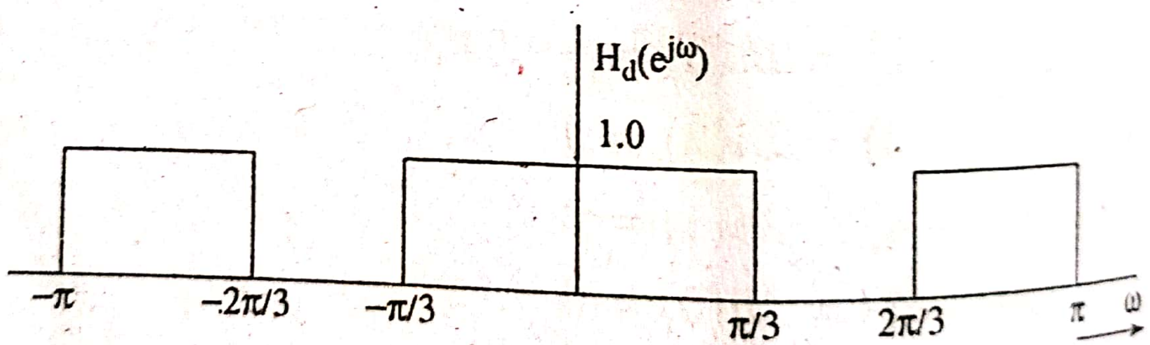

Fig. 6.14 Frequency response of Bandreject filter of example 6.8.

We know

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}\,d\omega$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-2\pi/3} e^{j\omega n}\,d\omega + \int_{-\pi/3}^{\pi/3} e^{j\omega n}\,d\omega + \int_{2\pi/3}^{\pi} e^{j\omega n}\,d\omega\right]$$

$$= \frac{1}{2\pi jn}\left[e^{-j2\pi n/3} - e^{-j\pi n} + e^{j\pi n/3} - e^{-j\pi n/3} + e^{j\pi n} - e^{j2\pi n/3}\right]$$

$$= \frac{1}{\pi n}\left[\sin\pi n + \sin\frac{\pi}{3}n - \sin\frac{2\pi}{3}n\right] \qquad -\infty \le n \le \infty$$