Example 3.20 Find the output $y(n)$ of a filter whose impulse response is $h(n) = \{1, 1, 1\}$ and input signal $x(n) = \{3, -1, 0, 1, 3, 2, 0, 1, 2, 1\}$ using (i) overlap-save method (ii) overlap-add method. (Annamalai Unverisity Apr' 03)

Solution

(i) Overlap-save Method

The input sequence can be divided into blocks of data as follows.

$$x_1(n) = \{\underbrace{0, 0}_{M-1=2 \text{ Zeros}} \quad \underbrace{3, -1, 0}_{L=3 \text{ data points}}\}$$

$$x_2(n) = \{\underbrace{-1, 0,}_{\text{Two datas from previous block}} \quad \underbrace{1, 3, 2}_{\text{3new data points}}\}$$

$$x_3(n) = \{3, 2, 0, 1, 2\} \text{ and } x_4(n) = \{1, 2, 1, 0, 0\}$$

given $h(n) = \{1, 1, 1\}$

Increase the length of the sequence to $L + M - 1 = 5$ by adding two zeros.

$$\text{i.e. } h(n) = \{1, 1, 1, 0, 0\}$$

$$y_1(n) = x_1(n) \textcircled{N} h(n) = \{-1, 0, 3, 2, 2\}$$

$$y_2(n) = x_2(n) \textcircled{N} h(n) = \{4, 1, 0, 4, 6\}$$

$$y_3(n) = x_3(n) \textcircled{N} h(n) = \{6, 7, 5, 3, 3\}$$

$$y_4(n) = x_4(n) \textcircled{N} h(n) = \{1, 3, 4, 3, 1\}$$

Note: Circular convolution of the sequences left as an exercise to the students.

$$\boxed{\underbrace{-1, 0}_{\text{discard}}, 3, 2, 2}$$

$$\boxed{\underbrace{4, 1}_{\text{discard}}, 0, 4, 6}$$

$$\boxed{\underbrace{6, 7}_{\text{discard}}, 5, 3, 3}$$

$$\boxed{\underbrace{1, 3}_{\text{discard}}, 4, 3, 1}$$

$$y(n) = \{3, 2, 2, 0, 4, 6, 5, 3, 3, 4, 3, 1\}$$

(ii) Overlap-Add method

Let the length of data block be 3. Two zeros are added to bring the length to five $(L + M - 1 = 5)$.

Therefore,

$$x_1(n) = \{3, -1, 0, 0, 0\}$$
$$x_2(n) = \{1, 3, 2, 0, 0\}$$
$$x_3(n) = \{0, 1, 2, 0, 0\}$$
$$x_4(n) = \{1, 0, 0, 0, 0\}$$

$$y_1(n) = x_1(n) \textcircled{N} h(n) = \{3, 2, 2, -1, 0\}$$

$$y_2(n) = x_2(n) \textcircled{N} h(n) = \{1, 4, 6, 5, 2\}$$

$$y_3(n) = x_3(n) \textcircled{N} h(n) = \{0, 1, 3, 3, 2\}$$

$$y_4(n) = x_4(n) \textcircled{N} h(n) = \{1, 1, 1, 0, 0\}$$

$$\boxed{3, 2, 2, -1, 0}$$

↕ ↕ add

$$\boxed{1, 4, 6, 5, 2}$$

↕ ↕ add

$$\boxed{0, 1, 3, 3, 2}$$

↕ ↕ add

$$\boxed{1, 1, 1, 0, 0}$$

$$y(n) = \{3, 2, 2, 0, 4, 6, 5, 3, 3, 4, 3, 1\}$$

Example 3.21 Using linear convolution find $y(n) = x(n) * h(n)$ for the sequences $x(n) = (1, 2, -1, 2, 3, -2, -3, -1, 1, 1, 2, -1)$ and $h(n) = (1.2)$. Compare the result by solving the problem using (a) overlap-save method (b) overlap-add method.

Solution

The linear convolution of $x(n)$ and $h(n)$ is

$$y(n) = x(n) * h(n) = \{1, 4, 3, 0, 7, 4, -7, -7, -1, 3, 4, 3, -2\}$$

Overlap-save method

The input sequence can be divided into blocks of data as follows.

$\longrightarrow$ $M - 1$ zeros appended

$$x_1 = \{0, \underbrace{1, 2, -1,}_{\text{3 datas}} \}$$

$$x_2(n) = \{-1, \underbrace{2, 3 - 2}_{\text{3 new datas}}\}$$

$\longrightarrow$ $M - 1 = 1$ data from previous block

$$x_3(n) = \{-2, -3, -1, 1\}; \quad x_4(n) = \{1, 1, 2, -1\}; \quad x_5(n) = \{-1, 0, 0, 0\}$$

Given $h(n) = \{1, 2\}$. Appending two zeros to the sequence we obtain

$$h(n) = \{1, 2, 0, 0\}$$

$$y_1(n) = x_1(n) \textcircled{N} h(n)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 2 \\ 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 4 \\ 3 \end{bmatrix}$$

$$y_2(n) = x_2(n) \textcircled{N} h(n)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 2 \\ 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} -5 \\ 0 \\ 7 \\ 4 \end{bmatrix}$$

Similarly $y_3(n) = \{0, -7, -7, -1\}$; $y_4(n) = \{-1, 3, 4, 3\}$;
$y_5(n) = \{-1, -2, 0, 0\}$

$\boxed{-2, 1, 4, -3}$
↓
discard

$\boxed{-5, 0. 7, 4}$
↓
discard

$\boxed{0, -7, -7, -1}$
↓
discard

$\boxed{-1, 3, 4, 3}$
↓
discard

$\boxed{-1, -2, 0, 0}$
↓
discard

$$y(n) = \{1, 4, 3, 0, 7, 4, -7, -7, -1, 3, 4, 3, -2\}$$

Overlap-add method

In this method the sequence $x(n)$ can be divided into data blocks as shown below.

$$x_1(n) = \{1, 2, -1, 0\}$$

↓ $M - 1 = 1$ zero added

$$x_2(n) = \{2, 3, -2, 0\}; \quad x_3(n) = \{-3, -1, 1, 0\}$$

$$x_4(n) = \{1, 2, -1, 0\}; \quad h(n) = \{1, 2, 0, 0\}$$

$$y_1(n) = x_1(n) \text{ (N) } h(n) = \{1, 4, 3, -2\}$$

$$y_2(n) = x_2(n) \text{ (N) } h(n) = \{2, 7, 4, -4\}$$

$$y_3(n) = x_3(n) \text{ (N) } h(n) = \{-3, -7, -1, 2\}$$

$$y_4(n) = x_4(n) \text{ (N) } h(n) = \{1, 4, 3, -2\}$$

| 1, 4, 3, −2 |
↕ add

| 2, 7, 4, −4 | $(\because M - 1 = 1)$
↕ add

| −3, −7, −1, 2 |
↕ add

| 1, 4, 3, −2 |

$$y(n) = \{1, 4, 3, 0, 7, 4, -7, -7, -1, 3, 4, 3, -2\}$$

Example 3.22 The linear convolution of length-32 sequence with a length-1000 sequence is to be computed using (a) overlap save method and (b) overlap-add method. The circular convolution of the impulse response sequence with each data block is to be obtained using DFT and IDFT method with $N = 64$. Find the number of DFTs and IDFTs required to obtain $y(n)$ for both methods.

Solution Given; $L_s = 1000; M = 32; N = 64$

(a) overlap save method

In this method since $(M - 1) = 31$ data points are lost we pad the data sequence with 31 zeros. The length of data sequence $L_s + M - 1 = 1031$.

For $N = 64$, the convolution of $h(n)$ with data block results in $64 - 31 = 33$ correct values.

The total number of DFTs required for the data are $\left\lceil \frac{1031}{33} \right\rceil = 32$.

One DFT is required to find DFT of impulse response. Hence the total number of DFTs is 33 and IDFTs is 32.

(b) overlap-add method $N = 64$

In each data block $M - 1 = 31$ zeros appended. Hence each data block has $64 - 31 = 33$ datas. Thus the total number of DFTs required is $\left\lceil \frac{1000}{33} \right\rceil = 31$.

One DFT is required for impulse response.

The total number of DFTs required is $31 + 1 = 32$

The total number of IDFTs required is 31.