FIG.11.3
Equivalent circuit of Fig.11.2

From the circuit $V_\pi = I_3 \times r_\pi$

The current source $g_m V_\pi$ in parallel with resistance R_L can be replaced with a voltage source $g_m V_\pi R_L$ in series with R_L using Thevenin's theorem. The resistors R' and r_π can be combined to form a single resistance $R = (R' + r_\pi)$. The final equivalent circuit is shown in Fig.11.4. Writing loop equations for the three loops,

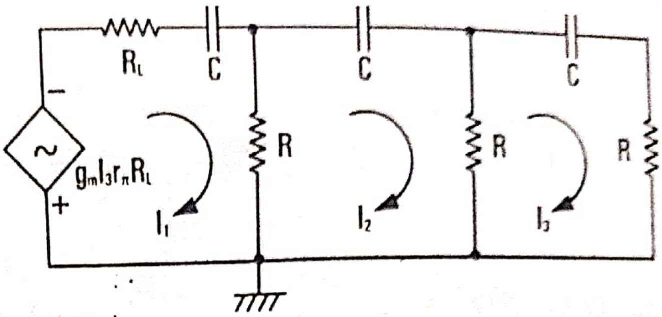

FIG.11.4
The final equivalent circuit

$$g_m r_\pi I_3 R_L + I_1 R_L + \frac{I_1}{j\omega C} + (I_1 - I_2)R = 0 \tag{11.3}$$

$$(I_2 - I_1)R + I_2 \times \frac{1}{j\omega C} + (I_2 - I_3)R = 0 \tag{11.4}$$

and

$$(I_3 - I_2)R + I_3 \times \frac{1}{j\omega C} + I_3 \times R = 0 \tag{11.5}$$

But $\quad g_m r_\pi = \beta$

The Eqs. (11.3), (11.4) and (11.5) can be written in matrix form as

$$\begin{bmatrix} R_L + R + \frac{1}{j\omega C} & -R & \beta R_L \\ -R & 2R + \frac{1}{j\omega C} & -R \\ 0 & -R & 2R + \frac{1}{j\omega C} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = 0 \tag{11.6}$$

Since the loop currents I_1, I_2 and I_3 cannot become zero, we have

$$\begin{vmatrix} R_L + R + \frac{1}{j\omega C} & -R & \beta R_L \\ -R & 2R + \frac{1}{j\omega C} & -R \\ 0 & -R & 2R + \frac{1}{j\omega C} \end{vmatrix} = 0 \quad (11.7)$$

Solving the determinant,

$$\left(R_L + R + \frac{1}{j\omega C}\right)\left[\left(2R + \frac{1}{j\omega C}\right)^2 - R^2\right] + R\left[-R\left(2R + \frac{1}{j\omega C}\right) + \beta R_L R\right] = 0$$

$$\text{or } R^3 + 3R^2R_L - \frac{R_L}{\omega^2C^2} - \frac{5R}{\omega^2C^2} + \beta R^2R_L + \frac{6R^2}{j\omega C} - \frac{1}{j\omega^3C^3} + \frac{4RR_L}{j\omega C} = 0 \quad (11.8)$$

when the imaginary part of Eq.(11.8) becomes zero, there remain only the real part at that frequency. i.e. the phase shift around the loop becomes zero. This frequency corresponds to the frequency at which the oscillator oscillates. Equating imaginary parts of Eq.(11.8) on both sides,

$$\frac{6R^2}{\omega C} - \frac{1}{\omega^3C^3} + \frac{4RR_L}{\omega C} = 0$$

$$\text{or,} \quad \omega^2C^2 = \frac{1}{6R^2 + 4RR_L} \quad (11.9)$$

1.7
ien-bridge oscillator (a) Using BJT (b) Using FET

ysis

he 360⁰ phase shift required for the oscillator is provided by two e amplifier circuits. Hence only RC networks should be considered nalysis. The feedback network is shown in Fig.11.8. First consider series and parallel RC networks. From the circuit,

$$V_f = \frac{V_o \dfrac{R\times 1/j\omega C}{R+1/j\omega C}}{R+1/j\omega C+\dfrac{R\times 1/j\omega C}{R+1/j\omega C}}$$

$$= \frac{V_o \times \dfrac{R}{1+j\omega RC}}{\dfrac{1+j\omega RC}{j\omega C}+\dfrac{R}{1+j\omega RC}}$$

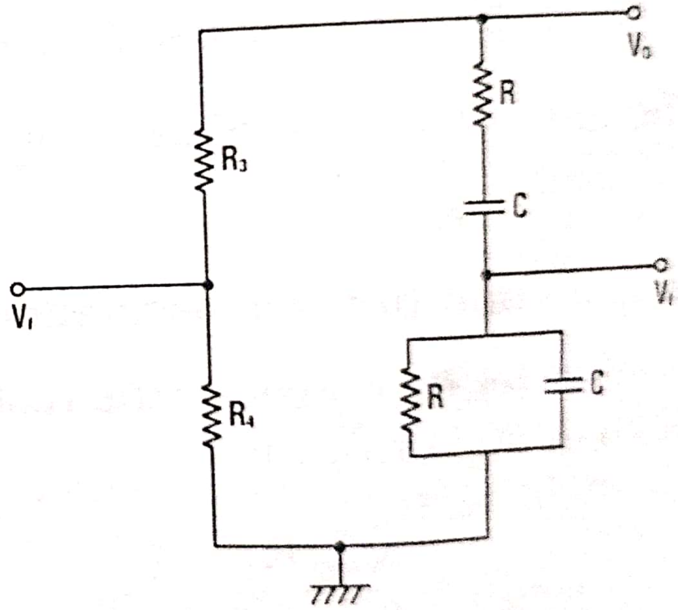

FIG.11.8
Feedback network

$$V_f = \frac{V_o \times j\omega RC}{(1 + j\omega RC)^2 + j\omega RC} = j\frac{V_o \omega RC}{1 - \omega^2 R^2 C^2 + 3j\omega RC}$$

or,

$$\frac{V_o}{V_f} = \frac{1 - \omega^2 R^2 C^2}{j\omega RC} + 3 \qquad (11.25)$$

Equating the imaginary part of Eq.(11.25) on both sides,

$$0 = \frac{1 - \omega^2 R^2 C^2}{\omega RC}$$

i.e. $\omega^2 R^2 C^2 = 1$

or, frequency of oscillation

$$f = \frac{1}{2\pi RC} \qquad (11.26)$$

FIG.11.9
Hartley oscillator (a) Using BJT (b) Using FET

Analysis

Replacing the BJT by its small signal model, the circuit of Fig.11.9(a) reduces as shown in Fig.11.10. FET oscillator circuit also reduces as in Fig.11.10 except that in place of r_π, it is an open circuit. In the circuit, R include transistor output resistance r_0 and load resistance of the oscillator circuit.

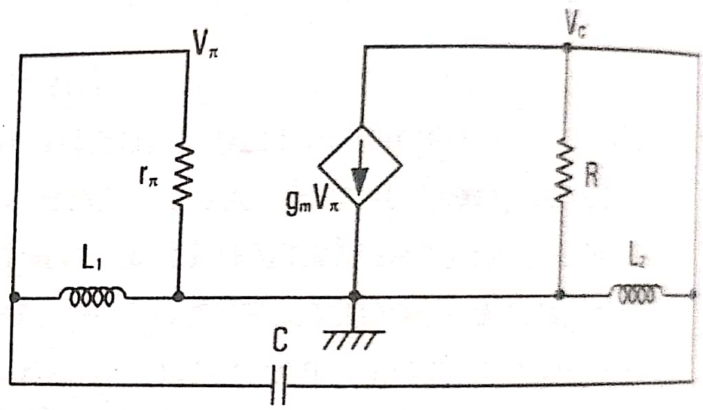

FIG.11.10
Equivalent circuit of Fig.11.9(a)

Writing mode equations

$$g_m V_\pi + \frac{V_C}{R} + \frac{V_C}{sL_2} + (V_C - V_\pi)sC = 0 \qquad (11.31)$$

The Eqs. 11.30 and 11.31 can be written in matrix form as

$$\begin{bmatrix} \frac{1}{r_\pi} + \frac{1}{sL_1} + sC & -sC \\ g_m - sC & \frac{1}{R} + \frac{1}{sL_2} + sC \end{bmatrix} \begin{bmatrix} V_\pi \\ V_C \end{bmatrix} = 0 \qquad (11.32)$$

Since node voltages V_π and V_C cannot be zero,

$$\begin{vmatrix} \frac{1}{r_\pi} + \frac{1}{sL_1} + sC & -sC \\ g_m - sC & \frac{1}{R} + \frac{1}{sL_2} + sC \end{vmatrix} = 0 \qquad (11.33)$$

Solving the determinant,

$$j\frac{\omega C}{r_\pi} + j\frac{\omega C}{R} + \frac{1}{j\omega L_1 R} + \frac{1}{j\omega r_\pi L_2} + jg_m\omega C + \frac{1}{Rr_\pi} - \frac{1}{\omega^2 L_1 L_2}$$

$$+ \frac{C}{L_1} + \frac{C}{L_2} = 0 \qquad (11.34)$$

Equating the real parts of Eq. 11.34 on both sides, we get

$$\frac{1}{R r_\pi} + \frac{C}{L_1} + \frac{C}{L_2} = \frac{1}{\omega^2 L_1 L_2}$$

$\frac{1}{R r_\pi}$ is negligible.

Hence, $$\omega^2 = \frac{L_1 L_2}{L_1 L_2 (L_1 C + L_2 C)}$$

Or $$\omega^2 = \frac{1}{L_1 C + L_2 C} \qquad (11.35)$$

Or, frequency of oscillation $f = \frac{1}{2\pi\sqrt{(L_1 + L_2)C}}$ (11.36)